Math Trailblazers Research and Revision Study
Annual Report to the National Science Foundation
Year 4

PART I. INTRODUCTION

The *Math Trailblazers* Research and Revision Study is an examination of the *Math Trailblazers* curriculum and its impact on teaching and learning. Its purpose is to inform revision of the curriculum, as well as contribute to the general knowledge of the impact of comprehensive, *Standards*-based mathematics curricula in elementary schools. It is composed of four research projects outlined below. This report describes the current status of each of the studies, outlines their implications for revisions to the curriculum, and provides information on the status of the revisions process.

The **Implementation** and **Whole Number Studies** are being conducted at the Institute for Mathematics and Science Education (IMSE) at the University of Illinois at Chicago (UIC). The **Implementation Study** is a systematic investigation of the implementation of the curriculum in K-5 classrooms. The **Whole Number Study** focuses on students' conceptual and procedural knowledge of whole number concepts. The following researchers are working on these studies:

- Catherine Randall Kelso, Co-Director, TIMS Project
- Stacy Brown, Visiting Research Assistant Professor
- Susan Beal, Visiting Research Professor
- Catherine Ditto, Teacher on Loan, Chicago Public Schools
- Cheryl Kneubuhler, Program Associate
- Kathleen Pitvorec, Program Associate
- Kelly Rivette, Research Assistant, Department of Mathematics, Statistics, and Computer Science
- Philip Wagreich, Professor, Department of Mathematics, Statistics, and Computer Science
- Reality Canty, Research Assistant, Department of Psychology, Cognitive Division
- Sara Atkins, Research Assistant, College of Education
- Richard Coppola, Research Assistant, College of Education
- John Sparks, Research Assistant, Department of Information and Decision Sciences, College of Business Administration
- Jennifer Bay-Williams, Associate Professor, Department of Elementary Education, Kansas State University

The **Video Study** is an analysis of the teaching and learning processes in *Math Trailblazers* classrooms. Researchers videotape and analyze first-grade lessons on place value and fourth-grade lessons on fractions. This research is being conducted by:

- Lucia Flevares, Assistant Professor, School of Teaching and Learning, Ohio State University
- Michelle Perry, Professor of Educational Psychology, University of Illinois at Urbana Champaign

The **Fraction and Proportionality Study** is an evaluation of students' learning of fractions and proportionality in grades 3-5. The following researchers from the Department of Curriculum and Instruction in the College of Education at the University of Minnesota are conducting this study:

- Kathy Cramer, Associate Professor
- Terrence Wyberg, Lecturer

PART II. THE WHOLE NUMBER AND IMPLEMENTATION STUDIES

The team responsible for the Whole Number and Implementation Studies is working on the two investigations simultaneously. The research questions are similar and much of the data are being analyzed together to provide a larger data set. In general, data for grades K-2 were collected during the 2003-04 school year and data for grades 3-5 were collected during 2004-05. See Appendix A for an outline of the research design of each study that was included in last years' report. Appendix A also includes tables that show the number of schools and classrooms in each study and provides demographic information for each school. This report will be organized around the following research questions that are addressed by these studies:

A. Which revisions to the curriculum are necessary in order to better support teachers' use of the curriculum?
   1. What components of the curriculum do teachers use?
      a) What units and lessons do they use or omit?
      b) How are lessons modified? Supplemented?
   2. How do teachers use the curriculum?
      a) What are the characteristics of a high fidelity classroom?
      b) What are the characteristics of a low fidelity classroom?
      c) How do teachers expand and enrich the curriculum?
   3. What factors' influence teachers' use?

B. What revisions to the curriculum are necessary in order to better support students' learning?
   1. To what extent are *Math Trailblazers'* students developing mathematics concepts and operations?
   2. To what extent are students' understandings related to their experiences with the *Math Trailblazers* lessons?

Which revisions to the curriculum are necessary in order to better support teachers' use of the curriculum?

What components of the curriculum do teachers use?
To answer this question, teachers in both studies completed surveys. Teachers in the Whole Number Study completed written surveys for each semester and teachers in the Implementation Study completed electronic surveys for each unit in their grades. Tables 1 and 2 show the number of teachers in each study and the number of each type of survey collected.

Table 1. Surveys Collected for the Whole Number Study

Grade	# of classrooms that completed the study	# of teachers who withdrew	# of Students	Fall Written Surveys	Spring Written Surveys
K	5	4	67	5	5
1	6	1	92	7	6
2[a]	12	1	172	12	10
3	8	1	126	8	7
4	7	1	125	6	6
5[b]	8	0	177	3	4
Total	46	8	759	41	38

[a]Data collected over two years (2003-04 & 2004-05) [b]Data collected over two years (2004-05 & 2005-06) and is ongoing

In order to be able to draw from a more robust data set, we asked four second-grade teachers to participate in the Whole Number Study for a second year. Two of these teachers were new to the curriculum in 2003-04 and two were long-term users. Collecting data from their classrooms again in 2004-05 will allow us to look at the changes that occurred in the new teachers' classrooms from their first to the second year of use in comparison to changes in classrooms of the long-term users. For similar reasons, we returned to two fifth-grade classrooms in 2005-06 and added a new teacher to that cohort.

Table 2. Surveys Collected for the Implementation Study

Grade	# of classrooms that completed the study	# of teachers who withdrew	# of Students	Fall Written Surveys*	Electronic Surveys
K	9	2	127	10	71
1	11	4	151	13	83
2	6	5	118	8	47
Sp Ed	1	1	5	1	4
3	5	6	61	1	68
4	6	5	101	0	54
5	6	3	38	1	81
Total	44	26	601	34	408

*Teachers who joined the study late or preferred not to use a computer completed written surveys for some or all the units.

Of the 134 teachers who originally consented to take part in the studies, 34 or 27%, withdrew. The rate of completion for the surveys was better in some grades than others. For example in grade 5, all the teachers that completed the Implementation Study completed either a written survey for the entire year or electronic surveys for each of the 16 units. However, in grade 4 the average number of electronic surveys completed per teacher was only nine (out of a possible 16). To increase the completion rate for grades 3, 4, and 5, we announced at the feedback meeting in August that we would increase the payment for each electronic survey. We continued to send reminders to teachers and collect surveys until March of 2006. Similar incentives were offered to teachers in the Whole Number Study to complete written surveys, but many still did not complete them.

Data from the two studies is entered into the same database for each grade. Data entry and analysis of the K-2 surveys is nearly complete. Similar work for grades 3-5 is under way. Survey data has given us a rich source of information that will help us develop a framework for revising each grade and for revising individual lessons.

<u>Which lessons and units do teachers use or omit</u>? Figure 1 shows a graph of lesson usage by unit for grade 1. For a given unit, unit usage is 100% if all teachers reporting on a unit used all the lessons in the unit. For example, the unit usage[1] for Unit 1 Grade 1 is 98% because every teacher used all the lessons in the unit, except one teacher who omitted one lesson.

[1] Unit Usage = (total number of lessons used by all teachers) ÷ (# of lessons in unit x number of teachers reporting)

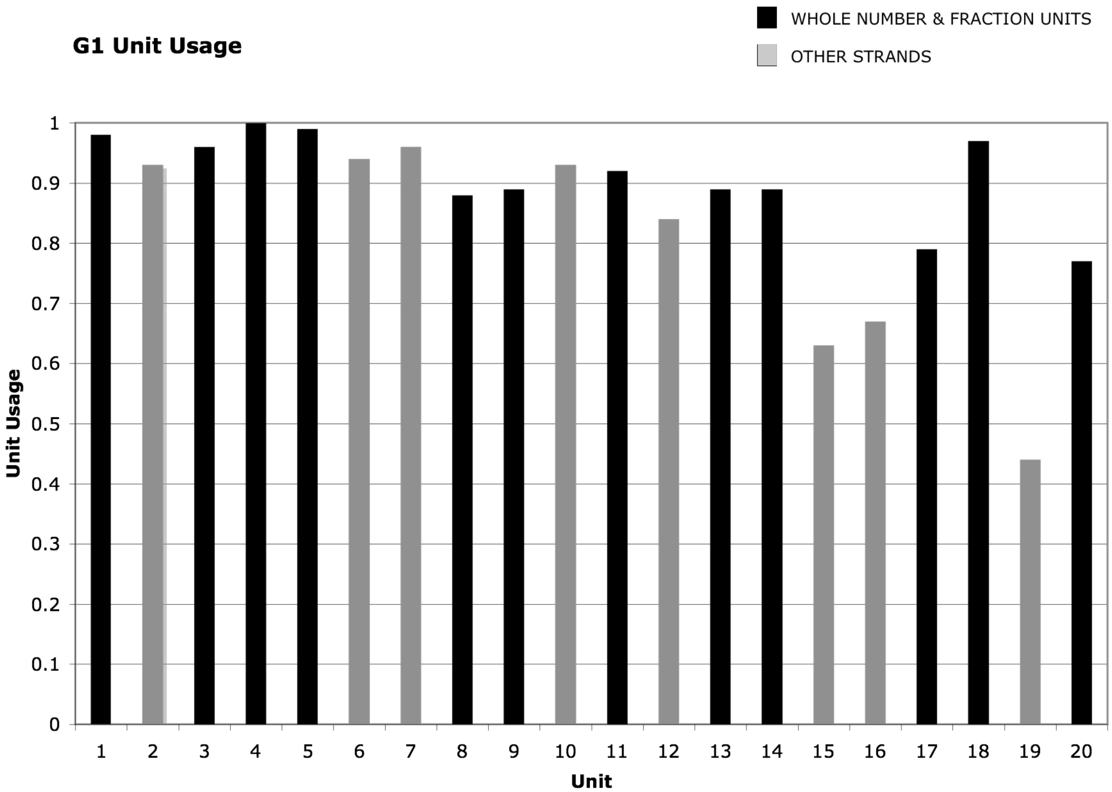


Figure 1. Grade 1 Unit Usage

In both grades 1 and 2, teachers tended to complete lessons in the number strand. If they chose to omit lessons, they omitted lessons from other strands. For example, in grade 1 Units 2, 16, and 19 are geometry units and Unit 15 is a data unit. Unit 7 is a unit on length, but students measure distances by grouping and counting links in a chain, which is an important component of the number strand. This data is similar to that reported by Tarr, et al. who reported that middle-grades teachers completed units on number more often than other strands and that placement of content strands across a grade did not affect teachers' choices of content to include or omit (Tarr, et al., 2006).

<u>How are lessons modified? Supplemented</u>? In general, teachers listed lessons as particularly successful more often than unsuccessful.[2] Teachers' reasons for modifying lessons or supplementing the curriculum fell into three broad categories: more practice, increased mathematical "pay off", and alignment with state and local standards.

Teachers asked for more practice–on both skill development and opportunities for students to consolidate concepts. In the Classroom Observation Protocol, we define this as providing "situations in which students may refine their applications of concepts, strategies, or operations; or develop their repertoire of strategies or operations by applying them to problem situations." (See Appendix B for a sample Classroom Observation Protocol.) We can use information from the classroom observation analyses that documents the ways that teachers create these situations in classrooms along with teacher suggestions from the surveys to provide activities and explicit suggestions in the materials that will create these opportunities.

[2] For example, grade 2 teachers listed a lesson as successful more often than unsuccessful for 77% of the lessons.

Survey data and classroom observations have provided us with information on lessons that do not provide enough mathematical pay off for the amount of time and energy it takes to develop the context of the lesson. For example, students may spend more time cutting and pasting than learning about multiplication. If we see a lesson with more beans on the floor than good strategies described in discussion, then the lesson needs revision. On the other hand, if teachers report that students use the representations to reason about mathematics or successfully use tools to solve problems, then those lessons should not be changed dramatically. Here are three quotes from surveys that provide guidance for revising a lesson on multiplication in which students make multiple mice from toothpicks and yarn. Taken together these comments tell us that if we change the lesson so that students simply draw the mice, then the lesson will provide more mathematical pay off.

> "Making Mice was hard to do because it was difficult to get the toothpicks, yarn, etc., to stay glued and then you needed to wait until the next day when the glue was dried to draw the features. HOWEVER, the children loved having the mice."

> "The children really loved this lesson and it got them thinking multiplicatively. They discovered how to use skip counting to make predictions for the number of parts on six mice."

> "We drew the mice parts in different colors instead of putting them together."

Teachers also reported that they modified or supplemented the curriculum to meet their state and local standards. For this reason, we are conducting a detailed analysis of standards documents from thirteen key states designated by our publisher. This review coordinates the state standards with the National Council of Mathematics *Principles and Standards for School Mathematics* (NCTM, 2000) and *Math Trailblazers'* scope and sequence documents. These analyses combined with reviews of documents summarizing state standards documents from all states (Lott & Nishimura, 2005; Reys, et al., 2006) provide guidelines for omitting or adding content to the new edition.

May 2007 Annual Report Insert: What components of the curriculum do teachers use?

The grade 3 survey data from both the Whole Number and Implementation Studies have been compiled and analyzed in the following ways using procedures similar to those used for the grades 1 and 2 data:

- We have developed a spreadsheet that gives the percent of teachers that completed each lesson, the percent of teachers that considered each lesson particularly successful in terms of student learning, and the percent that considered each lesson particularly unsuccessful.
- We have compiled teacher comments from the surveys for each lesson and unit. These comments are in response to questions that ask why teachers considered lessons to be particularly successful or unsuccessful in terms of student learning. They also include information on which lessons teachers find hard to implement, that is, which lessons did not have enough mathematical "payoff" for the time and energy that it takes to develop the context of the lesson. General comments about the units provide information on why teachers chose to complete some units and lessons and not others.

End May 2007 Annual Report Insert: What components of the curriculum do teachers use?

How Do Teachers Use the Curriculum?

To understand how teachers implement the curriculum in their classrooms, the research team collected the following data: videotapes of classroom observations and records of teacher interviews that preceded and followed the observations. Tables 3 and 4 show the number of each type of data collected.

Table 3. Implementation Study: Classroom Observations and Pre- & Post-Interviews

Grade	Classrooms	Students	Observations	Pre- and Post-Interview Sets
K	9	127	10	9
1	11	151	10	19
2	6	118	10	10
Sp Ed	1	5	1	1
3	5	61	4	4
4	6	101	6	6
5	6	38	4	4
Total	44	601	45	53

Table 4. Whole Number Study: Classroom Observations and Pre- & Post-Interviews

Grade	Classrooms	Students	Observations	Pre- and Post-Interview Sets[c]
K	5	67	9	8
1	6	92	13	12
2[a]	12	172	31	22
3	8	126	26	14
4	7	125	18	14
5[b]	8	177	24	15
Total	46	759	121	85

[a]Data collected over two years (2003-04 & 2004-05) [b]Data collected over two years (2004-05 & 2005-06) and is ongoing
[c]Researchers often conducted one set of interviews for more than one observation.

During the past year, three members of the research team have concentrated their efforts on refining the classroom observation protocol and using it to analyze the classroom observations for grades 1 and 2. Each classroom was observed one to three times during implementation of whole number lessons. Pre-observation questions asked teachers about the big ideas of the lesson, planned activities for the session, and anticipated difficulties that students might encounter. Post-observation questions asked them to reflect on their responses in the pre-observation interview and often to answer interviewer questions on choices they made as they taught the lesson.

The rationale for development of the observation protocol comes from recommendations, such as Shoenfeld's, for evaluation of curriculum implementation in which "data gathering, coding, and analysis must try to indicate the character of the implementation and its fidelity to intended practice" (Shoenfeld, 2006). In December of 2005, during a consultation with James Hiebert on our research design, he suggested that we avoid conflating fidelity to the literal curriculum (explicit instructions) and the "elaborated" curriculum (opportunities to learn) that may only be implicit ideas in the curriculum. His advice was to clearly separate the following two issues:

1) Did the teacher follow the literal description of the lesson?
2) What were the opportunities to learn?

Based on these recommendations and our previous work on the Classroom Observation Protocol, we chose to define and consider three curricular forms in the analysis of *Math Trailblazers* lessons as they occurred in classrooms:

- the *literal curriculum* that consists of the instructional materials provided to teachers, i.e., textbooks and teacher guides;
- the (*Math Trailblazers*) *intended curriculum* or the curriculum discerned from applying the stated philosophical approach to the mathematical content as articulated in the lesson; and
- the *enacted curriculum* that consists of the "opportunities to learn" mathematics that actually occur as teachers and students engage in the lesson.

Considerable time and thought were needed to develop and refine the definitions for the opportunities to learn that are included in the revised protocol. They reflect the goals and ideas of the *Principles and Standards for School Mathematics* (NCTM, 2000) and the curriculum's philosophical approach (Wagreich, et al., 2004). Appendix B provides complete definitions of the opportunities to learn, a blank protocol document for evaluating the lesson observations, and a sample from a completed protocol. The opportunities to learn are divided into two major categories as shown below:

a) Opportunities to explore, use, and deepen mathematical knowledge
   1) Reason to solve problems; reason about a mathematical concept
   2) Use or apply concepts, strategies, or operations; refine strategies so that they become more efficient
   3) Select from multiple tools, representations, or strategies
   4) Compare and make connections across tools, representations, or strategies
   5) Validate strategies or solutions; reason from errors; inquire into the reasonableness of a solution

b) Opportunities to communicate about mathematics
   1) Communicate mathematical ideas or ways of reasoning
   2) Interpret another student's way of reasoning about tools, representations, strategies, or operations
   3) Clarify or justify reasoning or explanations
   4) Characterize mathematical operations

The revised protocol for evaluating the observed lessons contains two major components: an evaluation of fidelity to the literal lesson and an evaluation of the fidelity to the intended lesson. To operationalize the above definition of the literal curriculum, researchers designed the section for evaluating the literal lesson in a two-column format. In the first column, researchers create an outline of the explicit recommendations and information to teachers provided by the authors in the Lesson Guide. The level of alignment between the observed lesson and the literal lesson is indicated in the second column. Researchers indicate whether the setup, procedure, and discussion points written in the lesson were implemented, partially implemented, or not implemented. Finally, they designate the level of fidelity to the literal lesson as low, moderate, or high. Appendix B includes a blank copy of the Literal Lesson Evaluation and excerpts from a completed protocol including a sample evaluation.

Researchers use the remaining sections of the protocol to evaluate the observed lesson in order to characterize the level of fidelity to the intended lesson. This is a three-step process:

- Describing the key mathematical foci of the lesson;
- Coding the enacted lesson for opportunities to learn; and
- Comparing the enacted lesson with the intended curriculum.

Two team members divide the transcript of a lesson into segments. Independently, they code the segments for the appropriate opportunities to learn, limited opportunities to learn, or missed opportunities. They note when any of the key mathematical content is addressed during the segment. These codes are then transferred to the Classroom Observation Protocol. Researchers evaluate the level of fidelity to the intended lesson by examining the codes and looking for themes–opportunities that consistently arose or were missed during the observed lesson. This rating can be low, moderate, or high to indicate the extent to which the observed lesson aligned with the intended lesson.

Each classroom observation is then placed on the Fidelity Grid according to the independent ratings of the literal lesson and intended lesson as shown in Table 5. Appendix B includes a blank Classroom Observation Protocol and Fidelity Grid, and excerpts from a completed protocol.

Table 5. Fidelity Grid for Grade 1

		LITERAL		
INTENDED		LOW	MEDIUM	HIGH
	LOW	C5	C2	C2
	MEDIUM		C4	C4 C5
	HIGH		C6	C1, C1 C3

Preliminary Findings

Analysis of grade 1 and 2 classrooms showed variability within sites. That is, within schools where teachers had similar classroom settings (demographics, materials) and similar professional development, observed lessons varied in terms of fidelity to the intended and literal lessons. For example, in a school where students and teachers were new users and where teachers attended district-wide professional development together, classroom observations within the school varied in the level of fidelity to the intended curriculum and literal lesson. These classrooms are shown as C1 and C2 on the Fidelity Grid in Table 5. (Classrooms with two observations are listed twice.)

The table also shows variability within classrooms. That is, the level of fidelity to the intended lesson is not indicative of the level of fidelity to the literal lesson and vice versa. This claim is illustrated in the example above as well as by several of the grade 1 and grade 2 lesson observations. These findings indicate that fidelity is not an attribute of schools or districts, but of individual lessons as implemented in classrooms. It depends on how the curriculum lives in the classroom with students and teachers. This variablity also indicates that teachers play a critical role in the implementation of curricula, for even within similar classroom settings, there is variability in the levels of fidelity.

February 2007 Update: How Do Teachers Use The Curriculum?

The classroom observation team has been concentrating on completing the analysis of the grade 2 classroom observations in order to understand how teachers use the curriculum. They have streamlined the analytic process and selected 24 of the 30 videotaped lessons in the Whole Number Study to analyze. Since the focus of the analysis is on understanding fidelity to the intended curriculum, the team has chosen multiple observations of specific content areas in order to look across enactments of particular concepts (addition and subtraction, place value, multiplication and division). For example, the team is looking at lessons in which students use base-ten pieces to represent subtraction. These observations may inform the revision of lessons that involve the use of base-ten pieces and reach beyond the context of subtraction to include recommendations for lessons that involve base-ten pieces in the teaching of place value and addition as well. The team is on track to complete the analysis of the selected grade 2 observations by the end of February in order to make recommendations to the writing team for revisions to the grade 2 whole number strand.

May 2007 Annual Report Insert: How Do Teachers Use the Curriculum?

The team of researchers analyzing the second-grade classroom observation data wrote a document in early March outlining procedures, findings, and recommendations for revisions. Using the classroom observation protocol described above, the team analyzed 20 classroom observations and produced the Fidelity Grid in Table 5A. The entries in the grid are for observations in the classrooms of five teachers as represented by the codes at the beginning of the entries[3].

Table 5A. Fidelity Grid for Grade 2

Intended \ Literal	Low	Moderate	High
Low		212 (PastaPV2 1104)[a] 215 (BT Sub1 0205) 215 (Zoo K2 0204) 215 (Zoo Stamp 0204)	211 (PastaPV1 0105) 212 (BT Sub1 0204) 212 (BT Sub2 0204) 212 (BT Sub1 0205) 215 (BT Sub2 0205) 215 (Zoo K1 0204) 215 (Zoo Lunch 0305) 215 (Pasta PV1 1104)
Moderate			213 (Pasta PV1 0105) 213 (BT Sub2 0304) 212 (BT Sub2 0205)
High		211 (Z Lunch 0504) 214 (Zoo St 0404)	213 (BT Sub1 0304) 214 (Add S 0204) 214 (P&P 0304)

[a] Each entry includes the teacher code (abbreviated lesson name and month and year of observation).

[3] The grade 2 observations took place over two years (2003-04 and 2004-05) as noted in Table 4. Four of the five teachers shown on the fidelity grid in Table 5 were observed both years and therefore represent eight of the classrooms in Table 4. Three other classrooms in Table 4 are not included in the sample in the grid because they were not observed during appropriate whole number lessons or teachers were absent due to maternity leave.

Preliminary Findings

The level of fidelity to the literal curriculum is on the horizontal axis and proceeds from left to right, low to high. Note that a majority of the enactments fell in the column farthest to the right indicating that the majority of enactments had a high level of fidelity to the literal curriculum. The level of fidelity to the intended curriculum is on the vertical axis and proceeds from top to bottom, low to high. Note that a majority of the enactments fell in the top row indicating that the majority of enactments had a low level of fidelity to the intended curriculum.

The Fidelity Table shows that there does not appear to be a relationship between fidelity to the literal curriculum and fidelity to the intended curriculum. Of the 15 enactments that were rated high fidelity to the literal curriculum, only 3 of them were rated as high fidelity to the intended curriculum as well. One pattern seen in this table is that there is some consistency for individual teachers' enactments with regard to the level of fidelity to the intended curriculum. Individual teachers tended to have either low-to-moderate or moderate-to-high fidelity to the intended curriculum over multiple enactments.

Table 5A indicates how well the overall opportunities to learn in each enactment align with the intended curriculum. It does not provide information about the two dimensions of fidelity to the intended curriculum—reasoning and communication. Therefore, the research team used a grounded theory approach to develop summary categories for the reasoning and communication opportunities to learn. They defined the summary categories by describing the degree to which the specified opportunities can be expected to arise for each related code. Table 5B shows the summary categories for the first three communication codes (B1, B2, B3). Researchers developed a similar table for the reasoning codes (A codes).

Table 5B. Communication Codes Describing the Range of Fidelity from Low to High

Communication Codes / **Student Contribution Type: Low to High Fidelity**	**B1** (describe ways of reasoning about tools, representations, strategies, or operations)	**B2** (interpret another student's ways of reasoning about tools, representations, strategies, or operations)	**B3** (clarify or justify reasoning or explanations)
Highly Structured, Limited Student Contributions	Hit/ Limited/ Missed Teachers ask numerous fill-in-the-blank questions and set simple expectations for descriptions. If students are asked to describe their ways of reasoning, the question posed is often, "How did you solve it?" Student responses tend to be short. Sometimes the teacher will guide students through a longer description with fill-in-the-blank questions, or the teacher may even complete descriptions for them. These opportunities, if they appear in the protocol, are sometimes limited or missed.	None Observed Students interpreting, responding to, explaining, or questioning each other's thinking, strategies, representations, and solutions does not tend to appear in these protocols.	Missed/ None Observed If there are occasions where clarification would make sense in the context of the lesson activities (e.g., students share strategies or work together to problem solve), these opportunities generally do not appear in the protocol, or they appear as missed opportunities where a clarifying question could have been asked but was not.
Limited Student Contributions	Hit/ Limited Teachers ask numerous fill-in-the-blank questions and set simple expectations for descriptions. When students are asked to describe their ways of reasoning, the question posed is often, "How did you solve it?" Student responses tend to be short. Sometimes the teacher will guide students through a longer description with fill-in-the-blank questions. In the protocol these opportunities are sometimes limited.	None Observed Students interpreting, responding to, explaining, or questioning each other's thinking, strategies, representations, and solutions does not tend to appear in these protocols.	Limited/ Missed When clarification makes sense during the lesson (e.g., students sharing strategies), these opportunities may appear in the protocol, but will generally be limited as the teacher may ask for clarification, but the student still does not provide a complete response, or the teacher guides the student's response with fill-in-the-blank questions or by finishing the description for the student. Some missed opportunities may also appear where a clarifying question could have been asked but was not.

Communication Codes, con't / **Student Contribution Type: Low to High Fidelity**	**B1** (describe ways of reasoning about tools, representations, strategies, or operations)	**B2** (interpret another student's ways of reasoning about tools, representations, strategies, or operations)	**B3** (clarify or justify reasoning or explanations)
Supported Student Contributions	Hit Teachers ask students to describe their ways of reasoning, with questions that begin with *how* or *why*. Student responses are sometimes short, but sometimes students provide more complete descriptions in their responses. The teacher may guide students through a longer description with prompts or questions. In the protocol these opportunities are rarely limited.	Hit/ Limited/ Missed Teachers create situations so students are expected to talk to each other about their thinking, representations, and strategies (e.g., "explain to your partner how" or "describe your drawing to your partner"). Teachers invite students to respond to each other's thinking or ask questions about each other's work, encouraging them to listen to and build off of each other's ideas. These opportunities may be limited as the responses are directed toward a solution rather than a process or idea. The teacher may intercede in an interaction so these opportunities are limited or missed.	Hit/ Limited Teachers ask students to clarify their descriptions or to justify their strategies, representations, or ideas. When student responses are incomplete, the teacher might continue to probe with prompts and questions. Sometimes students are seen asking the teacher for clarification. These opportunities may be limited in that students do not give complete clarifications and justifications, and the teacher does not prompt them further or may finish the description for students.
Rich Student Contributions	Hit Teachers ask students to describe their ways of reasoning, with questions that begin with *how* or *why*. Students often provide more complete descriptions in their responses. The teacher may guide students through a longer description with prompts when necessary. In the protocol these opportunities are almost never limited.	Hit Teachers create situations where students are expected to talk to each other about their thinking, representations, and strategies (e.g., "explain to your partner how" or "describe your drawing to your partner"). With and without the teacher's invitation, students respond to, question, and correct each other's thinking in whole-class, small-group, and partner situations. Students listen to and build off of each other's ideas. In the protocol these opportunities are rarely limited.	Hit or None Observed Often, student responses are complete, so that no clarification or justification is required. When it is, teachers probe students for further clarification or justification with prompts and questions. Students may ask teachers for clarification. These opportunities may not appear in abundance in the protocol due to students' proficiency with their descriptions.

Each enactment was placed in a summary category for reasoning (A codes) and a summary category for communication (B codes) based on the codes that were identified in the enactment and recorded in the protocol. The enactments were then displayed in the Reasoning and Communication Table (Table 5C) on both dimensions according to the summary categories.

Table 5C. Reasoning and Communication Summary Table.

Type of Learning Environment Established for Reasoning / Student Contributions to Discourse	Observe-Practice-Repeat Learning Environment (Teacher-led Learning Environment)	Minimal Student-Participation Learning Environment	Limited "Community of Inquiry" Learning Environment (Limited Student-Participation Learning Environment)	"Community of Inquiry" Learning Environment (Co-constructed Learning Environment)
Highly-Structured Discourse, Limited Student Contributions	**212 (BT Sub1 0204)** **212 (BT Sub2 0204)** **215 (BT Sub1 0205)** **215 (BT Sub2 0205)** **215 (Pasta PV1 1104)** **215 (Zoo K1 0204)** **215 (Zoo K2 0204)** **215 (Zoo Stamp 0204)** **215 (Zoo Lunch 0305)**	**212 (BT Sub1 0205)** **212 (BT Sub 2 0205)**		
Limited Student Contributions	**211 (Pasta PV1 0105)**	**212 (Pasta PV2 1104)** **213 (Pasta PV2 0105)**	213 (BT Sub2 0304)	
Supported Student Contributions				213 (BT Sub1 0304)
Rich Student Contributions			211 (Z Lunch 0504) 214 (Add S 0204) 214 (P&P 0304)	214 (Zoo St 0404)

The horizontal dimension represents a progression in opportunities to reason. It moves from left to right, lowest-to-highest fidelity to the intended curriculum. At the low end (to the left), opportunities to reason tend to be more teacher-led or teacher-structured. At the high end (to the right), opportunities to reason tend to be more co-constructed by teachers and students.

The vertical dimension represents a progression in opportunities to communicate. It moves from top to bottom, lowest-to-highest fidelity to the intended curriculum. At the low end (the top row), teachers tend to guide communication with fill-in-the-blank or known-answer questions. At the high end (the bottom row), students tend to communicate with the teacher and each other in more open-ended ways.

The shading on the table represents the overall level of fidelity to the intended curriculum. The top-left quadrant (light shading) tends to include the enactments having a low level of fidelity to the intended curriculum and the lower-right quadrant (dark shading) tends to include the enactments having a high level of fidelity to the intended curriculum.

In thinking about how the two dimensions work together in the top-left quadrant (related to a low level of fidelity to the intended curriculum), students tend to observe, practice, and repeat what the teacher introduces, explains, and demonstrates. The teacher often scaffolds students' problem solving so that there is little student exploration of procedures, mathematical concepts, or ideas. This tends to reduce opportunities for students to make decisions about their course of action and to generally reduce the mathematical complexity of questions, dialog, and tasks. Most communication is between the teacher and students, and it is often structured with fill-in-the-blank and known-answer questions.

In the lower-right quadrant (related to a high level of fidelity to the intended curriculum), there is a high level of student autonomy. Students tend to make decisions in their problem solving, choose the strategies they will use, and evaluate their work considering the reasonableness of their strategies and solutions. The communication that occurs often includes rich student dialog. The teacher uses open-ended questions and prompts to invite students to clearly and completely explain and justify their thinking. Students listen to, respond to, question, and comment on the teacher's and classmates' ideas. They have opportunities to compare their strategies and build off of each other's ideas.

From inspection of Table 5C, there appears to be a correlation between the overall levels of reasoning and communication. For instance, if an enactment is rated high for reasoning (placed to the far right), then it also tends to be rated as high for communication (placed in the bottom row). All of the enactments high on one axis, are high on both axes and are therefore placed in the lower-right quadrant. This correlation is not surprising considering how opportunities for students to choose, compare, and make connections across strategies might support richer opportunities for them to interpret other students' thinking or to describe and clarify their strategies and solutions.

Another pattern that emerges from the table is that lessons for which enactments are consistently placed in the top-left quadrant appear to be inherently non-supportive of a high level of fidelity to the intended curriculum regardless of who the teacher is. There is little difference between teachers' enactments in terms of level of fidelity to the intended curriculum. For example, the four enactments of the *Pasta Place Value* lesson only occur in the top-left quadrant. The instructions provided to teachers in these lessons–the literal lessons–have particular characteristics. They tend to specify procedures for teachers to explain or demonstrate and for students to repeat while allowing little opportunity for student exploration. In the absence of student investigation, there is little for students to reason about, in terms of selecting and comparing strategies or responding to others' interpretations or approaches. Opportunities to learn may actually be constrained when teachers implement these lessons with a high level of fidelity to the literal curriculum.

Other lessons, such as *Base-Ten Subtraction* have their enactments distributed across categories and quadrants. One difference between these lessons and the lessons described previously is that, when categorized by teacher, there is a significant difference between teachers' enactments along both dimensions. Teachers with high-level fidelity enactments for other lessons, tend to have high-level fidelity enactments for these lessons. Likewise, teachers with low-fidelity enactments tend to have low-fidelity enactments for these lessons. This observation indicates that particular lessons may simultaneously support opportunities to learn that align with the intended curriculum while at the same time allowing low-fidelity enactments. Reviewing these lessons in the curriculum shows that they do not tend to script exactly what the students should do. They often include less-structured instructions. Although sometimes the lessons specify procedures for teachers to demonstrate, these procedures tend to build on initial student exploration providing opportunities for reasoning and communication. Teachers may interpret the instructions as formulas for "teaching" students step-by-step procedures, which may constrain the opportunities to learn in an enactment. Or, teachers may interpret the instructions as a framework for engaging students in an investigation of ideas and concepts, which may provide opportunities for reasoning and communicating.

A third observation from the table involves enactments in the lower-right quadrant, the quadrant related to a high level of fidelity to the intended curriculum. Few teachers consistently make it into the highest levels for the reasoning and communication categories. In fact, in second grade, the enactments for only one teacher are all located in the lower-right quadrant. This suggests that is difficult to consistently enact lessons so that they have a high degree of student autonomy, co-constructed opportunities to learn, and rich student dialog. For example, in considering the communication dimension, researchers noted that although many enactments contained whole-class discussions where students shared strategies, the level of communication often did not progress to the higher categories of *Supported* or *Rich Student Contributions*. Teachers tended to guide the discussion asking students to respond to structured questions or comments, and little student-to-student dialog occurred.

In summary, the analysis of classroom observations has prompted the team to think about what information can and should be included in lessons to make the intended curriculum more transparent—that is, to better align the literal curriculum with the intended curriculum. These observations will be revisited in Part VI Revision of the Curriculum in the discussion of implications for revision.

End May 2007 Annual Report Insert: How Do Teachers Use the Curriculum?

What factors influence teachers' use?

To answer this question, researchers have collected data from the following sources:

- Interview and survey questions concerning the duration and form of professional development,
- Interview questions concerning on-site support, surveys and videotaped discussions concerning teachers' beliefs about the weaknesses and strengths of the units and lessons,
- Pre-observation questionnaires and post-observation interview questions concerning teachers' decisions and classroom practices,
- Surveys and videotaped discussions concerning teachers' evaluations of video segments and characterizations of an ideal implementation, and
- Videotaped discussions concerning teachers' evaluations and characterizations of student work.

Of particular interest are data collected at the summer feedback meetings. The purpose of the meetings was to gather additional data on teachers' use of the curriculum materials and to collect data on teachers' beliefs. At the meetings, participants completed a written survey that amplifies and clarifies the existing survey data in the focus areas of pacing and meeting the diverse needs of students. For example, participants were asked to list what kinds of additional materials they believed would "practice, extend, or deepen the content" for students.

At these meetings, participants took part in two sessions in which, on computers, they viewed a videotape of a *Math Trailblazers* lesson at their grade level and commented on the lesson. The purpose of the lesson reviews was to tap participants' "ideal scripts" for lessons that embody the teaching and learning principles of reform mathematics (Jacobs & Morita, 2002). Participants recorded mathematical issues that they saw arise as students engaged in the lesson, as well as how they saw students using strategies and skills. They recorded evidence of students' understandings and misunderstandings and how they saw the classroom teacher addressing these issues. In addition, participants described how they envisioned the ideal implementation of the lesson. After their computer sessions, participants engaged in videotaped discussions of their responses to the lessons.

Finally, participants reviewed and evaluated several student work samples from each of two different problems at their grade levels. For each set of student samples, participants commented on the mathematics apparent to them in the problem, where they anticipated challenges for students, how they expected a typical student to solve the problem, and on what criteria they would evaluate student work for each type of problem. Tables 6 and 7 summarize the number of Lesson Reviews and Problem Reviews collected to date.

Table 6. 2003-2004 Problem and Lesson Reviews

Grade	Number of Problem Reviews	Number of Lesson Reviews
K	26	26
1	19	20
2	16	16

Table 7. 2004-2005 Problem and Lesson Reviews

Grade	Number of Problem Reviews	Number of Lesson Reviews
2	5	4
2/3	0	13
3	9	8
4	6	16
5	8	15

As described in last year's report, researchers have completed some preliminary work categorizing participants' written comments for the lessons. It is anticipated that the coding scheme will continue to evolve as analysis of this rich data set continues. The information gathered from the summer feedback meetings will provide further context for understanding the other data sources in this study.

What revisions to the curriculum are necessary in order to better support students' learning?

To what extent are *Math Trailblazers'* students developing mathematics concepts and operations?
To answer this question, the research team is collecting the data listed below. Table 8 shows the number of each type of data collected.

- Classroom observations
- Student interviews corresponding to Key Content from observed lessons (Whole Number Study only)
- Student work samples from observed lessons
- Additional student work samples, including *End-of-Year Tests*
- Student achievement data

Table 8. Number and Type of Data on Student Learning

Grade	Implementation Study Classrooms	Whole Number Study Classrooms	Students	Interviews[b]	Work Samples (Classroom Sets)
K	9	5	194	12	25
1	11	6	243	25	47
2[a]	6	12	290	67	56
Sp Ed	1	0	5	0	1
3	5	8	187	73	39
4	6	7	226	30	33
5[a]	6	8	215	64	28
Total	44	46	1360	271	229

[a]Data collected over a two-year period [b]Whole Number Study only

Analysis of Student Interviews. Three interviews were developed for each grade level that corresponded to the Key Content in the observed lessons from the whole number strand and the corresponding *Math Trailblazers* Assessment Indicators. One to three students from each classroom in the Whole Number Study were randomly selected to take part in the interviews. All of the grade 1 interviews and the majority of the grade 2 interviews have been coded.

The interviews for grade 1 are referred to as the Links, Cubes, or Graph Interview, according to the main tool used in each interview. The Links Interview explored students' knowledge of and abilities to group and count by tens, partition 100 into multiples of ten, apply relationships between addition facts and multiple of tens, and write number sentences for addition situations. The Cubes Interview explored students' strategies for solving word problems and their abilities to represent addition and subtraction situations with number sentences. The Graph Interview ascertained whether students could read a graph, use it to solve problems related to the display of data involving addition and subtraction, and write corresponding number sentences.

Of the thirteen students interviewed, eleven students took part in two interviews. These students were given the Links Interview and either the Cubes or Graph Interview. Table 9 shows students' performance on the interview tasks based on Key Content.

Table 9. Proficiency at Addressing Content in Interview Tasks

Content	Yes	No	No Evidence
Can students count objects by tens? (Links)	11	1	
Can students solve addition or subtraction problems involving multiples of ten? (Links)	10	2	
Do students make connections between basic addition facts for ten and multiples of ten? (Links)	5	3	4
Can students count on to solve addition problems? (Cubes)	5	1	1
Can students count on or back to solve subtraction problems? (Cubes)	6	1	
Can students interpret bar graphs? (Graph)	4	1	
Can students use data to solve problems? (Graph)	4	1	
Can students represent numbers using manipulatives and addition and subtraction situations using number sentences? (Links, Cubes, Graphs)	11	2	

Links (n = 12), Cubes (n = 7), Graph (n = 5)

To further analyze the student interview data, the team developed instruments based on previous research in the field to evaluate four dimensions of whole number understanding: reasoning (Carpenter, et al., 1999), flexibility (Dienes, 1960; Gray & Tall, 1994; Heibert & Carpenter, 1992; Spiro, et al., 1988), communication (Lane, 1993), and accuracy. These rubrics are shown below in Table 10. Appendix C includes copies of the rubrics with descriptions of possible student responses.

Table 10. Rubrics for Scoring Student Interviews

	0	1	2	3
Accuracy	Almost all correct responses	2 or more incorrect responses	1 incorrect, possibly two if many questions are asked	Responds correctly to all tasks
Reasoning	No meaningful strategy	Student understands the operation and must directly model the problem to solve it.	Student understands the operation and can apply counting to solve the problem.	Student can apply math-ematical reasoning about relations between numbers to accurately solve the problem.
Commun-ication	Explanation and/or description totally unclear or irrelevant; lacks supporting argument; use of symbols, tables, and graphs not present or completely inappropriate; does not use appropriate terminology	Explanation or description is possibly unclear (minimal); supporting arguments are incomplete or logically unsound; use of pictures, symbols, tables, and graphs are present, but with errors or are irrelevant; terminology used with major errors.	Explanation and or description is fairly complete and clear; supporting arguments are logically sound, but may contain minor gaps; use of pictures, symbols, tables, and graphs are present but with minor errors or somewhat irrelevant; terminology used with minor errors.	Explanation and or description is complete and clear; supporting arguments are strong and sound; use of pictures, symbols, tables, and graphs are correct and clearly relevant; terminology is clear, precise, and appropriate.
Flexibility	Not able to use a tool or representation to solve problems	Can use <u>one tool or representation</u> to model mathematical concepts and solve problems	Can use <u>multiple tools or representations</u> to model mathematical concepts and solve problems, but <u>does not make connections</u> across the representations	Can use <u>multiple tools or representations</u> to model mathematical concepts and solve problems <u>Recognizes connections</u> across representations

<u>Preliminary Results from Student Interviews</u>. Most students in the grade 1 sample demonstrated the ability to solve problems accurately. Figure 2 below is a graph that shows a positive relationship between accuracy and reasoning on the Links tasks. Similar graphical analyses show positive relationships between accuracy

and reasoning on the Cubes and Graphs tasks. However, reasoning scores were unstable across interviews as shown in Table 11. These results may be due to differences in the questions or tools used in the interviews. In terms of flexibility, nine of the 13 students used multiple tools to solve problems (scoring at a level 2 or 3 on the rubric), but only two of these students made connections between representations (level 3).

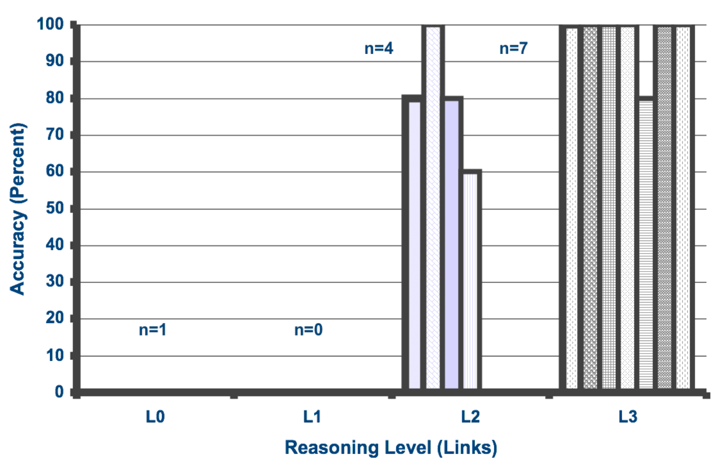
Data were combined for each student across interviews, yielding a “profile” or summary of what each student could do with respect to the tasks. We looked at the following: accuracy; reasoning level; manipulatives and strategies used to solve problems (number charts, fingers, cubes [counters], number sentences, mental arithmetic, graphs); and connections made between and among the manipulatives. See Appendix C for examples of profiles for two students.

These results point to further analysis that will identify the affordances and constraints associated with particular tools and specific mathematics situations. Currently the research team is applying rubrics adapted for use with the second-grade interview in their analysis of second graders’ performance.

Table 11. First grade: Summary of data over students and tasks

Class-room	Student #	Links				Cubes				Graphs			
		A	**R**	**F**	**C**	**A**	**R**	**F**	**C**	**A**	**R**	**F**	**C**
C1	101	0	0	0	1	0	0	0	0				
C1	102	3	3	3	3	3	2	3	3				
C3	103	2	2	2	3					3	2	3	3
C3	104	3	3	3	3					3	3	2	3
C3	105	3	3	1	3					2	2	2	2
C4	106	3	3	2	3	3	2	3	3				
C4	107	3	2	2	2	2	1	2	1				
C4	108	2	2	1	2	2	2	1	1				
C5	109	1	2	2	2								
C5	110	2	3	2	2					1	2	2	2
C2	111	3	3	3	2	3	2	2	2				
C2	112	3	3	2	3	3	2	2	2				
C5	113									0	0	0	2

Figure 2. Accuracy vs. Reasoning Levels on Links Interview



Analysis of student tests. We examined two other sources of data on first-grade students to insure that the interview data had meaning. The first source was the *End-of-Year Test* from the *Math Trailblazers* curriculum. Grade 1 teachers in both the Whole Number and Implementation Studies submitted a total of 143 tests. Analysis of the results of five items from the test that were similar to the tasks in the three interviews established that student performance on these five items was similar to performance in the interviews. The second source was the pre- and post-tests administered as part of the Video Study. We selected two questions from the Video Study tests that were similar to the Links Interview. The results from 105 students mirrored the results obtained in the Links Interview.

February 2007 Update: Analysis of Student Interview Data

All second-grade student interviews have been transcribed and coded. Thirty students were interviewed over two years. As not all students were interviewed with all protocols, the student interview team chose to analyze 46 interviews that addressed addition and subtraction. The rubrics in Table 10 were adapted for use with the second-grade tasks. Profiles were written that described performance on these tasks for each student. Each profile includes the following data: student number, gender, which interviews were given and number of questions asked, accuracy score for each interview, reasoning level for each interview, strategies and external representations used, connections made, performance on assessment indicators, and overall scores for accuracy, flexibility and reasoning level. The percentages met for each of the indicators are listed in the table.

Table 11A. Proficiency at Addressing Content in Grade 2 Interview Tasks

Assessment Indicator	YES	NO
Can student represent 2-digit addition problems using base-ten pieces?	83%	17%
Can student add multi-digit numbers using base-ten pieces, 200 Charts, pictures, paper and pencil, or calculators?	93%	7%
Can student solve addition problems and explain their reasoning?	87%	13%
Can student represent 2-digit subtraction problems using base-ten pieces?	86%	14%
Can student subtract 2-digit numbers using base-ten pieces, 200 Charts, pictures, paper and pencil, or calculators?	86%	14%
Can student solve subtraction problems and explain their reasoning?	80%	20%

The mean score for accuracy was 2.167(scale: 0–3), for flexibility the mean was 2.034, and for reasoning it was 2.033. These mean scores are higher than those for grade 1 on comparable dimensions.

All 11 interviews for Kindergarten have been transcribed. For third grade, 49 interviews have been transcribed and 12 have been coded. For fourth grade, 20 of the 30 interviews have been transcribed. For fifth grade, 9 of the 30 interviews have been transcribed.

The student interview team and classroom observation team are preparing to combine the data from their analyses in order to better understand the relationships between the opportunities to learn that are seen in the classroom and students' performance on the student interview tasks. This additional analysis will then be used in the revision of second grade.

May 2007 Annual Report Insert: Analysis of Student Interview Data

Coding of grade 3 student interviews and development of student profiles is nearly complete. Most grade 4 and grade 5 interviews are transcribed and ready for analysis. See May 2007 Insert for the following section for more information on student interview analysis.

End May 2007 Annual Report Insert: Analysis of Student Interview Data

To what extent are students' understandings related to their experiences with the *Math Trailblazers* lessons?

Data recorded in the Classroom Observation Protocols and in the Student Profiles provide an opportunity for qualitative analyses of the relationship between students' experiences in classrooms and their performance in interviews. In particular, we will look for relationships between students' opportunities to reason and communicate as recorded in the protocols with their success at reasoning and communication as recorded in their profiles.

The first column of Table 11 labels each student's scores with his or her classroom using the same numbering scheme for classrooms as in the Fidelity Grid in Table 5. The project's statistician has made a first attempt at a quantitative analysis of the relationship between fidelity of classroom observations and students' performance. Chi-square tests were examined to see whether student test scores were different for classrooms with different fidelity ratings. Due to the small numbers involved (13 students, 5 teachers) it is difficult to detect significant results. The only interview scores that showed a statistically significant result with intended fidelity scores were communication scores[4]. Since both the number of classrooms and number of students is larger for second grade, we will be able to conduct similar tests with larger sample sizes.

For his thesis, Reality Canty designed and implemented interviews on multiplicative reasoning for third- and fifth-grade students in the Whole Number Study. His thesis will be titled *Number Size, Structural Invariance, and Accuracy: Towards Understanding Children's Thinking in Multiplicative Situations.*

[4] These relationships were significant at alpha=0.10. Because the interview scores were ordinal and due to the small sample sizes involved, chi-square tests with Fisher's exact tests were used to assess statistical significance.

May 2007 Annual Report Insert: To what extent are students' understandings related to their experiences with the *Math Trailblazers* lessons?

To explore students' understandings in relation to fidelity ratings, grade 2 students were grouped by classroom. Table 11B shows the differences in scores on accuracy, communication, flexibility, and strategy (reasoning) from the profiles of the 21 students from the classrooms represented on the Fidelity Grid in Table 5A. The percent of students credited with the modal score is in parentheses. Cells with multiple entries indicate a bimodal distribution with the percent of students for both scores in parentheses.

Table 11B. Distribution of Modes for Student Scores Grouped by Level of Fidelity

Level of Fidelity to the Intended Curriculum	Accuracy	Communication	Flexibility	Strategy
low (n=11)	3 (54.55)	3 (45.45)	1, 2 (91.00)	2 (63.63)
high (n=10)	2 (60.00)	3 (50.00)	3 (60.00)	2, 3 (80.00)

The most striking differences in the chart are in the flexibility scores. In classrooms where we observed enactments that had low fidelity to the intended curriculum, a little over 90% of the students were able to use at least one external representation to support their thinking when problem solving, but these students were not able to make connections across the representations to attain a score of 3. On the other hand, in classrooms where we observed enactments that had high fidelity to the intended curriculum, a clear majority (60%) of students were able to use multiple external representations and make connections among them.

A specific task, the Marco Task, in one of the interviews serves as an example of the differences that can be seen in students' use of whole number operations. This task asked students to reason about another student's strategy for adding two-digit numbers.

Marco Task:

Propose the following scenario: “The other day I was talking to a boy named Marco and he was adding up his total ... he had some numbers … the problem was sixty-nine cents plus thirty-six cents.

Write:

$$\begin{array}{r} 69 \\ \underline{+36} \end{array}$$

“And, Marco did the following, he said sixty plus thirty is ninety ... so that’s ninety and nine plus six is fifteen.” Write

60+30 = 90
9+6=15

“After Marco said ‘sixty plus thirty is ninety and nine plus six is fifteen,’ he got confused. If Marco asked you to help him out what would you do?”

If the student does not understand the scenario or the questions you pose, try to rephrase the questions.

After the student has considered Marco’s approach ask, “How would you solve the problem?”

Student responses varied greatly. Figure 2A shows two student responses that exemplify the ends of the continuum–from a student who could only rely on the standard algorithm to a student who understood and could explain Marco’s strategy. Student 027 chose to solve the problem using the standard algorithm. Then, when probed by the interviewer to see if she could make a connection to Marco’s method, she simply replaced “Marco’s” representation with her own. Student 023 showed evidence that he understood Marco’s strategy and could unpack it and use it himself.

Student 027 Student 023

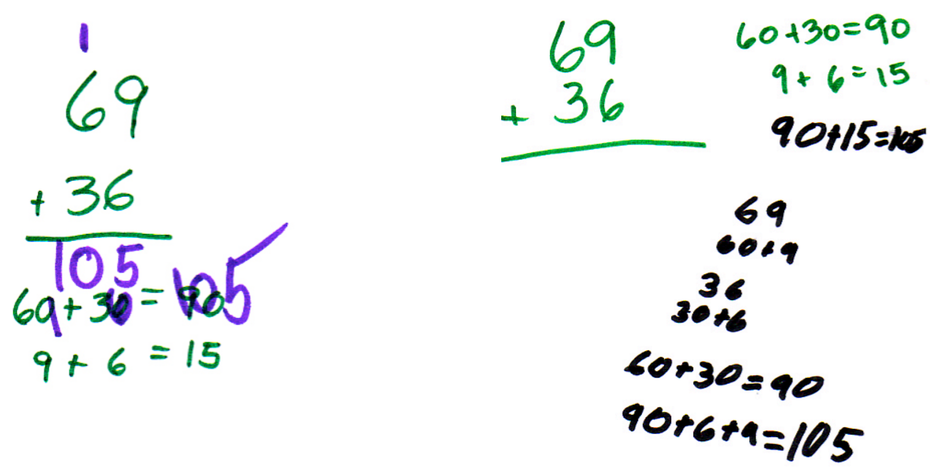


Figure 2A. Sample student responses to the Marco Task

Students' responses to this task were categorized as shown Table 11C. The table represents a total of 15 students[5] from the classrooms represented on the Fidelity Grid in Table 5A. Students from classrooms with a low level of fidelity to the intended curriculum are on the top half of the table. Students from classrooms in which we observed enactments that had a high level of fidelity to the intended curriculum are at the bottom.

Table 11C. Success on Marco Task by Level of Fidelity to the Intended Curriculum

Level of Fidelity (LOF)	T/S #	Help Marco				Solve on Own			
		Successful		Unsuccessful		Successful		Unsuccessful	
		Alt	Alg	Alt	Alg	Alt	Alg	Alt	Alg
Low LOF to Intended	215-026	X					X		
	215-027		X				X		
	215-111		X				X		
	212-005			X			X		
	212-025		X						
	212-102	X					X		
	212-112						X		
High LOF to the Intended	211-021		X						
	211-022	X				X			
	211-106	X				X			
	213-023	X				X			
	213-024					X			
	213-107	X							
	214-001	X							
	214-002		X			X			

An x in the first column denotes that the student was able to begin with Marco's representation and use it to "help" him solve the problem. An x in the second column means that the student helped Marco by using the standard algorithm. An x in the third column means that the student attempted using an alternate strategy to help Marco, but was unsuccessful.

The fifth and sixth columns show that all but 4 of the 15 students chose a successful strategy to solve the problem. An x in the fifth column represents the choice and successful use of a strategy other than the standard algorithm to solve the problem. These strategies included the use of base-ten pieces (moving both left to right and right to left), counting strategies, and a compensation strategy.

The table shows that only two of the seven students from classrooms in which we observed enactments that had a low fidelity to the intended curriculum were able to successfully help Marco using an alternative strategy. However, five of the eight students from classrooms in which we observed enactments that had a high level of fidelity to the intended curriculum were able to help Marco using an alternative strategy. Students from classrooms of low-fidelity enactments exclusively chose the

[5] All students did not take part in all interviews or tasks. Only 15 of the 21 students interviewed from the classrooms represented in Table 5A completed the Marco task.

standard algorithm and students from classrooms of high-fidelity enactments chose alternative methods.

Other interview tasks asked students to solve multi-digit addition and subtraction problems in more than one way. Inspection of students' performances on these computation tasks shows that all but three of the 15 students were able to successfully solve the problems using at least two strategies. Students chose tools and methods that included paper-and-pencil algorithms, base-ten pieces, *200 Charts*, and mental math strategies. The use of paper-and-pencil algorithms and the number and type of external representations did not vary much between the two groups of students. The successful use of paper-and-pencil algorithms was essentially the same for both groups. However, in the analysis of students' responses, researchers looked for evidence that students could make connections between their two representations of the operations. For example, some students made connections between their paper-and-pencil representations and the trades they made when solving the problem with base-ten pieces. There was evidence that all eight students from the classrooms with enactments that had a high level of fidelity to the intended curriculum made such connections. In the classrooms with low-fidelity enactments, researchers found evidence that only two of the seven students made similar connections. This observation supports the analysis of the flexibility data in Table 11B.

The analysis of the classroom observation data in relation to the student interview data has been problematic and therefore the findings are speculative. This is due to the small number of classroom observations that could be analyzed and the small number of students who could be matched with the classroom data. However, the analysis does suggest that there is a connection between the opportunities to learn students experience in their classrooms and their performance on interview tasks that involve the use of multiple strategies.

End May 2007 Annual Report Insert: To what extent are students' understandings related to their experiences with the *Math Trailblazers* lessons?

PART III. VIDEO STUDY

In the Video Study, we have collected video observations, student assessments, and teacher interviews. Analyses of both the first-grade data and fourth-grade data are under way. Results from the first-grade data are presented below.

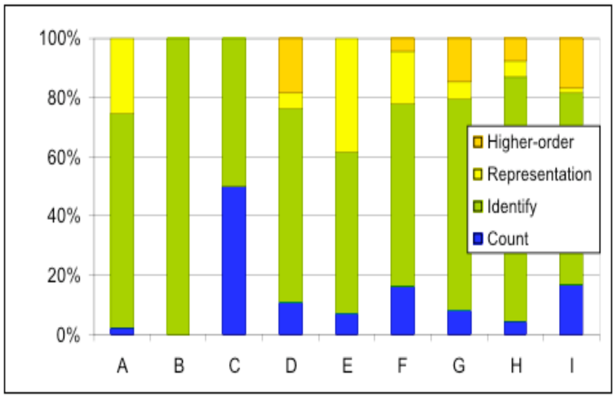
Lesson Analyses

To constitute a larger sample of study from the videos, we received four Grade 1 Unit 11 video observations from the Whole Number Study. As a result, analyses include nine lessons (Unit 11 Lesson 1 or 2). Teachers' classroom experience ranged from two to 29 years and their experience with *Math Trailblazers* ranged from one to four years. The student population was mostly urban, ethnically diverse, and middle to low income.

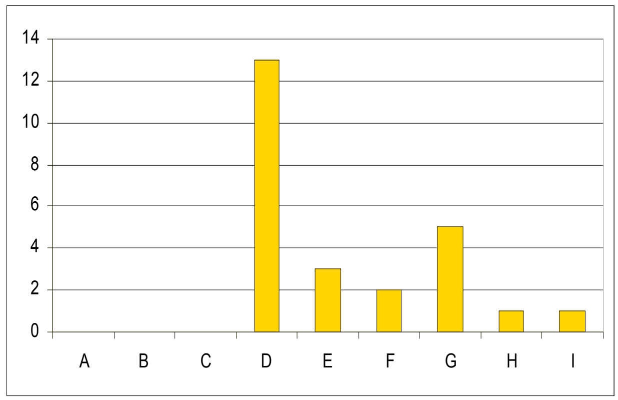
Discourse Analyses

In-progress analyses focus on teacher questions, teacher explanations, student explanations, and students' length of utterance. The *Math Trailblazers* teacher materials include some suggestions of questions for teachers to ask during the observed lesson and some guidance for discussion, but the curriculum does not heavily script the lessons, so teachers interpret lessons as they deem appropriate.

Teacher questions and explanations. Typical of elementary instruction, teachers asked a variety of questions during their lessons. For trends, we looked at the most common question types across teachers (request to count, identify, or recall; to describe a representation; or to offer an explanation or justification) and then looked for trends within those types. Figure 3 shows graphs of the proportions of the four most frequent types of questions and the rate of teachers modeling explanations.



Teacher Questions: Proportions of the 4 Most Frequent Types

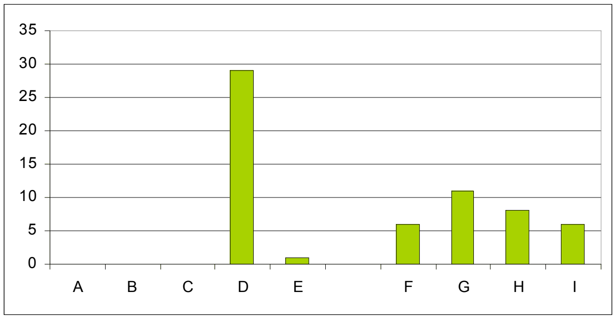


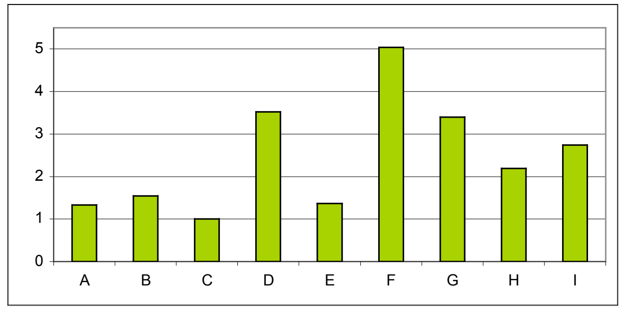
Teacher Explanations

Figure 3. Graphs of Most Frequent Types of Teacher Questions and Teacher Explanations

Of special interest are the most cognitively demanding questions, higher-order explanation and justification questions. Teachers differed in their use of these questions (x = 6.86% of whole-class questions, range 0 to 18.3%). Their use varied greatly by teacher: five teachers asked none, and the others asked these questions repeatedly, most notably Teacher D whose higher-order questions comprised almost one-fifth of all questions during whole-class time. Overall, teachers in the classes with the more faithful implementation of the curriculum asked questions like "Why did you say 70? You should have a reason." (Teacher E) and, "How else could you have figured that out? Does anyone know a different way?" (Teacher H) Teachers sometimes chose to model explanations for their students. Although most offered few (x = 2.78, range = 0 to 13), Teacher D, who asked students to explain most often, also gave the highest rate of explanations herself.

Student contributions. We found differences by classroom in the average number of words for a student turn (x = 2.46, ranging from 1.00 to 5.04 words per turn). Similarly, the average number of student explanations (x = 6.78, with a range of 0 to 29) includes some lessons with no explanations and others (especially Classroom D) where students articulated understandings of place value and representations well. See Figure 4.





Student Explanations **Average Number of Words Per Student Turn**

Figure 4. Graphs of Student Explanations and Length of Student Contribution

Errors and Confusion. We coded each of the nine lessons for the incidence of errors and confusion during whole-class instruction and found a range of 0 to 20 (x = 5.55). We found that the lessons with the discourse most reflective of the NCTM *Standards* also had the highest rates of student errors and confusion, offering opportunities to clarify student understanding and the simplest, most procedurally focused lessons had the fewest.

Representation Use. From initial analyses of the students' and teachers' use of base-ten representations we have found that the classrooms with more complex discourse also feature students in more hands-on, active roles in using and reflecting on multiple representations.

Student Group Work. Current analyses center on group-work within lessons, examining the student-student and any teacher-student discourse to understand the exchange of ideas, especially for students explaining and justifying their ideas (e.g. Cohen, 1994; Webb, 1991).

In summary, the following tended to occur together: more complex teacher-student discourse, more active student use of representations during whole-class time, and more errors and confusion, offering opportunities for public resolution. We observed a range of implementations of the observed *Math Trailblazers* lessons with some conducting the lessons in line with more traditional methods of instruction and others fully embodying the NCTM *Standards*. As we complete analyses in the coming months, the Video Study will continue to document demonstrated and potential ways that the curriculum can be implemented to best facilitate student thinking (Carpenter, Fennema, & Franke, 1996; Franke, & Kazemi, 2001).

PART IV. FRACTION AND PROPORTIONALITY STUDY

This study addressed the following questions:

- What have students learned about fractions and proportionality from the *Math Trailblazers* curriculum at grades 3, 4, and 5?
- To what extent have students met the fraction goals articulated in *Math Trailblazers* Assessment Indicators and in the NCTM *Standards* as outlined in the Grade 3-5 Number and Operation Strand?
- To what extent do Grade 5 students using the curriculum understand pre-proportionality ideas as articulated in previous work by the Rational Number Project (RNP)?

During the spring of 2005, the project staff at the University of Minnesota implemented the fraction and proportionality studies. We collected data on fourth- and fifth-grade students from three districts. The three districts were: Harvey, Illinois; St. Louis Park, Minnesota; and Dubuque, Iowa. The Illinois school has used *Math Trailblazers* for four years. The other two sites were in the second year of implementation. In Iowa we worked with two schools–one school involved fourth-grade students and the other school involved fifth-grade students. In Minnesota we worked with all grade 4 and 5 classrooms from two schools. We collected the data between May 16 and May 23, 2005.

The fourth-grade classroom teachers administered the written fraction test to all students in their classrooms. The fifth-grade classroom teachers randomly administered the same fraction test to half of their students and the proportionality test to the other half.

We randomly selected students for interviews from the total list of students who took the written tests. We interviewed 13 fourth graders and 33 fifth graders. Of the 33 fifth graders, 17 took part in the fraction interview while 16 took part in the proportionality interview.

Teachers also completed a detailed survey to document their use of the *Math Trailblazers* curriculum as it related to the fraction and proportionality units in the appropriate grades.

Fraction Data Analyses

During the 2005-06 academic year, project staff at the University of Minnesota focused on data analyses and reporting the results of the studies to the UIC group. From the teacher surveys we identified classrooms that completed some or all of the fraction units. Test and interview data reflect only those grade 4 classrooms in which the teacher taught at least Unit 12 of the two fraction units, 12 and 14. For grade 5, we identified those classrooms in which the teacher taught at least Units 3 and 5 from the following fraction units: 3, 5, 9, 11, and 12.

Table 12. Classrooms in Fraction Study

District	# grade 4 classrooms that completed at least Unit 12	# of grade 5 classrooms that completed at least Units 3 & 5
Illinois (Chicago)	2/2	2/2
Iowa (Dubuque)	2/2	4/4
Minnesota (SLP)	5/10 (4 from one school)	9/9

The test data and interview data were analyzed during the summer and fall of 2005. Students' answers were entered into the spreadsheet in one column; the next column identified if they were correct or not. When appropriate, student strategies were also identified. Test data were analyzed in several ways: (a) total test scores by grade level and totals by grade level and district; (b) individual test items by grade level and by district. Comparison data from previous fraction studies were included in the data summaries for the individual items.

Appendix D provides summary data for the fraction test by grade level for all students combined. The results on the fraction test were mixed, with students doing the best on the order items. Students' performance on the operations items was disappointing given that their performance on the order items suggest that they have an understanding of the relative size of fractions. Appendix D also includes sample data for an individual item with comparison data from the Rational Number Project study and another study involving the *Investigations* curriculum. This was done for all items on the fraction test (Cramer, et al., 2002).

Test information analyzed in various ways was compiled and presented to UIC staff in January of 2006. A sample from grade 5 showing one fraction interview item, students' responses, and interview notes is in Appendix D along with a summary chart for this item collapsing data across all grade 5 students.

The interviews were analyzed in detail to highlight key insights and summaries were constructed to highlight important ideas at the end of each interview section. For example, after analyzing the eight interview order items among grade 4 students, the following summary was reported:

The fraction chart is the model students use to construct mental representations for fractions. They rely on this fraction chart model to order fractions. Students did not refer to the pattern blocks at all when they ordered fractions. In contrast, RNP students relied on mental images of fraction circles to order fractions.

There seems to be some limitations to students making sense of fractions by using mental representations for the fraction chart. This can be seen in the difficulty they had with fractions not represented on the chart, fractions greater than one, and fraction pairs that lend themselves to the residual strategy. Perhaps something is missing with the fraction chart. It seems to support same numerator strategy but not the other strategies (transitive and residual). Perhaps it is because students cannot manipulate the fraction chart.

Proportionality Data Analyses

The two major units in grade 5 that deal explicitly with proportionality are Units 13 and 16. None of the grade 5 classrooms from the Minnesota site completed those units. The two grade 5 classrooms from Illinois covered Unit 13. None of the grade 5 classrooms in Iowa completed either of these units. These results then reflect what the students learned about proportionality from the curriculum as a whole, and not specifically related to Units 13 and 16.

Graduate students scored the written tests identifying correct, incorrect answers, and strategies. Kathleen Cramer analyzed the proportionality interviews and created a database for that data. A summary of the proportionality test data is in Appendix D. Some generalizations follow:

- Students were able to use tables and graphs to solve problems (80% or more)
- Missing value and numerical comparison problems with no integer relationships were difficult for all
- Numerical problems with integer relationships within measure spaces were easier than problems with integer relationships between measure spaces
- The more complex *Math Trailblazers* problem was difficult for all students

Interview data supported test results. Virtually all but two students interviewed were able to use tables and graphs to solve missing value problems. A sample of other insights reported to the authors follow:

<u>Missing Value Questions</u>. For the most part, students solved missing value problems by looking for the multiplicative relationship within a measure space. This is reflected in the high number of scale factor and equivalent ratio strategies whether they lead to correct or incorrect responses.

The following students' responses demonstrate how students focused on the within measure space relationship in Question 1 (4:10; ?:40).

Correct

- *I know that 4 pizzas can feed 10 students. So I go 10, 20, 30, 40. Then I know 4 pizzas can feed 10 students so I go. I multiply 4 times…wait a minute. Maybe 4÷4. That's 16 (she meant 4 x 4). 16 tells you how much you need. (student 10003)*
- *She's gonna order…she has to order 16 pizzas. She's got 10 students…than increased to 40…so she got from 10 to 40 by multiplying 10 times 4, so then I just multiplied 4 times 4, and it gave me 16. (student 10042)*

- *16 pizzas. I used proportions…my first is uh…4 pizzas is enough for 10 students, 8 pizzas is enough for 20 students, 12 pizzas are enough for 30 students, and 16 pizzas are enough for 40 students. (student 10038)*

Incorrect

- *You know that 4 pizzas is enough for 10 students…she needs 30 more pizzas because 10 is enough for 40…so she needs 30 more. and then…wait…I know…I goofed that up…she says 4 pizzas are enough for 10 students, so since 4 is enough for 10 then you just um…that goes into there 3 more times so 4 times 3 is…I want to make sure I am doing this right…12…so there's 12 pizzas. So that means that she knows that 4 pizzas is enough for 10, 4, 10, 30, times that by 3—that's 12. I: So does she need 12 pizzas or 12 more pizzas? S: She actually needs 12 total pizzas.*

Students' success with missing value problems dropped when the integer relationship within measure space was non-integer. This was true even when the integer relationship existed between measure spaces. Only three students could solve Question 2 (4:16; ?: 40). One used a cross product procedure. The other two correct solutions involved reducing 4:16 to 2:8 and using the within measure space integer relationship.

This reliance on integer scale factor is evident on Questions 3 and 4 that had no integer relationships. The only correct solution involved reducing the initial ratio so a scale factor could be used.

<u>Numerical Comparison</u>. Different strategies emerged from the numerical comparison tasks. Now we see students thinking in terms of the unit rate. For Questions 6 and 7, unit rate thinking and qualitative reasoning strategies were seen. For Question 8, equivalent ratios were used to answer the question. The numerical aspects may have influenced the students. In Question 8, students just needed to use a doubling strategy to compare the two ratios (4:88; 8:2.40). In the other questions, the numbers were not as "nice". Because of that, they turned to other strategies. Below are examples of unit rate thinking, qualitative reasoning, and equivalent ratios.

Unit rate

- *I am trying to figure out how many do each…how many do each caterpillar eat and how many each worm eats? I think each caterpillar eats…2 and 1/5, I mean 2…2 and ½. I: How did you figure that out? S: Cause…uh…one caterpillar…I made one of these in my head. This is one caterpillar, this is two…then one, two…so that's two pieces for each caterpillar and then there's one more. And it's two caterpillars in all so maybe 2 and ½, one of them got this half and one of them got this half. I: Okay. Should we work on the worms then? S: 1 and 1/3. I: So who eats more—the caterpillar or the worm? S: The caterpillar. (student 10038)*
- *This is another estimate. I'm thinking what times 4 equals 53. I know 12 x 4 equals 48. So' I'll try… Mateo because I both used estimates. I know what I need to multiply what Blanca had. The easiest number I could get to 42 grams is 40 5 x 8 equals 40. 5 x 9 equals 45. Mateo, I also did closest possible. That's 4 times 12 is 48.* I: How did you conclude his was bigger? S: *It was a higher number. (student 10003)*

Qualitative Reasoning

- *Um…I think they both eat…the caterpillar. There's only 2 caterpillars and it eats 5 leaves, and there's only 3 worms and it eats less leaves than 2 caterpillars. (student 20054)*
- *S: Mateo, because he had the 4 rocks that are 53 and she has 5 that are 52, and 5 is bigger than 4 so that makes … Mateo bigger because he has less and she has more that weighs less so he had the heavier rocks. (student 20061)*

Equivalent ratios

- *Jenna. I: Why did you decide Jenna got the better deal? S: She got 4…no…wait, wait, wait…yeah…Jenna. I: Can you just explain to me why you knew Jenna got the better deal? S: So Jenna, like if she wanted to buy 8 Snickers bars she would get that four $1.76 and his cost $2.40. That's right, so that's why I decided that. (student 10042)*

In January of 2006 the project staff from the University of Minnesota met with the project staff at UIC. We accumulated three notebooks of data summaries for the staff and spent two days reviewing the information.

The University of Minnesota group is currently working on a paper: *What Can We Learn From Students' Errors: An Examination of Students' Misunderstandings With Fraction Order, Estimation and Operations Tasks*. In this paper we examine students' errors that appear consistently among students using the *Math Trailblazers* curriculum and look to establish reasons for these errors. The errors reflect students' emerging understandings about fractions and are mostly conceptually based. We discuss curricula implications, and ways teachers can use these errors to guide their instruction.

Implications for revisions

1. Reconsider the concrete models used. The fraction chart model is the most salient model and does provide students with the visual support needed to understand the inverse relationship between the size of the denominator and size of the fraction part. However, the static nature of the chart does not seem to support students' abilities to combine fractional parts as they think about fraction estimation.
2. Since few students referred to the pattern block pieces in their thinking, reconsider using that model. Models should support the creation of mental images for fractions that help students make sense of order and estimation tasks. If the pattern blocks are not providing students with those images, then consider a different model. RNP recommends the circle model.
3. The dot paper model used in fifth grade for addition is a good one, but perhaps not the best one for students to use right away. The model closely matches the algorithm, which is one of its strengths. However, it may be too abstract for the initial model to use to add and subtract fractions. Again, RNP has found the circle model to be helpful to provide students with early experiences combining and separating fractional amounts. Students see concretely the need for common denominators when using the circles.
4. Consider including more fraction work in grade 4, since grade 5 includes so much.
5. Proportionality: students have developed some strong pre-proportionality ideas. As expected, fifth graders find tasks with non-integer relationships hard. However, *Math Trailblazers* students can and do use tables and graphs to solve problems involving proportional relationships. The one issue to address in revisions is the integer relationship between measure spaces. Students for the most part ignored these patterns that can be seen looking across rows in a table and relied almost exclusively on patterns going down the table.

PART V. OTHER ANALYSES OF THE CURRICULUM

Mathematics Content Review

During the summer of 2005 Tom Berger, Professor of Mathematics, Colby College, reviewed the content of *Math Trailblazers*, following the outline for content reviews of curricula laid out in the National Resource Council report, *On Evaluating Curricular Effectiveness: Judging the Quality of K-12 Mathematics Evaluations* (Confrey and Stohl, 2004). Professor Berger generally gave a very positive review of the clarity of objectives and mathematical accuracy in the curriculum. He "was strongly impressed by the mathematical accuracy from the point of view of mathematical organization, execution, and presentation. The ideas follow a sound logical development and are presented as intended: mathematically accurately. The text pays special attention to the accuracy of language."

Geometry Content Review

Linda Hallenbeck, working under the direction of Michael Battista, Professor of Teacher Education at Michigan State University, conducted the review of the geometry strand. She addressed the following areas: the clarity of objectives and their comprehensiveness of treatment; the accuracy and depth of mathematical inquiry and mathematical reasoning; the balance of curricular choices such as conceptual vs. procedural and use of context vs. decontextualized treatment; assessment; and lastly, teacher capacity and training, resources, and professional development (Confrey and Stohl, 2004).

Recommendations and comments from the review fall into three categories:

- Recommendations for modifying the lesson activities with the goal of enhancing the mathematical experience for students;
- Recommendations for enhancing teacher materials to improve teacher understanding and implementation of the lesson content; and
- Recommendations for enhancing the geometry content.

Her most frequent recommendations were for modifying the lesson activities with the goal of enhancing the mathematical experience for students. She submitted detailed analyses for each geometry lesson in grades 1 and 2. In general, the comments focused on:

- How teachers can support the development of the language of geometry to enhance the dialogue and enrich discussions. The consultant highlighted places in the lessons where prompts could be added.
- *Math Trailblazers* geometry lessons specify that students should use manipulatives. The consultant emphasized the importance of making manipulatives available for each of the student teams in all lessons, and not just for teacher demonstration models.
- The need for clear expectations for students. For example, it is important that students know the differences between two-dimensional and three-dimensional shapes.
- The use of discussions as closure to lessons. She suggested that the key ideas to be stressed should be highlighted for the teacher.

The recommendations for enhancing teacher materials to improve teacher understanding and implementation of lesson content included suggested short essay topics for the *Teachers Implementation Guide*. Examples include appropriate geometry expectations and experiences for students and a discussion about the research on children's learning about three-dimensional solids and volume. She also suggested notes for the Lesson Guides describing students' geometric misconceptions. Lastly, and perhaps most importantly, she listed many examples of questions that could be inserted as prompts for teachers to ask students as well as sample student responses.

There were a few recommendations for enhancing the geometry content. The report stated that clear expectations need to be set for students for each of the lessons. It also stated that the text should be enhanced with questions and discussion prompts for teachers to use to assess student thinking and to encourage student explanations. The journal prompts in *Math Trailblazers*, if made more explicit, could provide opportunities for students to explain their thinking. The consultant explained that the text should specify grade-level vocabulary that students need to know by the end of the year. However, it is clear that the consultant feels the lessons are appropriate and that they meet the NCTM Standards for K-2 Geometry, as well as the standards set by the State of Ohio.

May 2007 Annual Report Insert: Geometry Content Review

Linda Hallenbeck has agreed to review the geometry content for grade 3 using the same guidelines and procedures that we developed for grades 1 and 2.

End May 2007 Annual Report Insert: Geometry Content Review

PART VI. REVISION OF THE CURRICULUM

Implications for the Revisions Process

While the analysis of the research described above is ongoing, guidelines for the revision process described below have emerged from the work to date. These ideas come from the analysis of classroom observations from the Whole Number and Video Studies, student interviews from the Whole Number and Fraction and Proportionality Studies, student tests from all four studies, survey data from the Whole Number and Implementation Studies, current research from the field, consultant reports, and reviews of state and national standards.

Maintain successful curriculum components

Survey data indicate that teachers find the majority of lessons successful, student interview and test data indicate that grade 1 students can complete whole number tasks accurately, and classroom observation data show that lessons implemented with a high degree of fidelity provide students with the intended opportunities to learn. Results of the Fraction and Proportionality Study indicate that grade 4 and 5 students have an understanding of the relative size of fractions and have developed some strong pre-proportionality ideas. Comparison of the *Math Trailblazers* scope and sequence with state and national documents reveal that the content in the curriculum is appropriately aligned with state and national standards. The mathematician who reviewed the content was "impressed with the mathematical accuracy." These findings from a range of sources coupled with positive results from studies of achievement of *Math Trailblazers* students on high stakes tests[6] (Sconiers, et al., 2003; Carter, et al., 2003) provide evidence that the curriculum is successful in diverse classrooms. These results indicate that there are many successful components of the curriculum that should remain intact and that the authors should concentrate revisions on the components and lessons that, as indicated in the data, are not successful–those that are missing the mathematical pay off.

Improved Classroom Discourse

Analysis of the classroom observations from both the Whole Number and Video studies and recommendations from the geometry consultant indicate that teachers need information on how to facilitate richer discussions that develop and clarify students' understanding. Therefore, in the teacher materials, authors will include revised discussion prompts with possible student responses and sample student dialog, so teachers have a vision of rich discussion content and how to address student misconceptions through discussion. Materials will include explicit prompts for encouraging students to compare and evaluate each other's strategies. An important addition to the Lesson Guides will be a section that suggests appropriate ways to summarize lessons so that big ideas are reviewed and reinforced.

Review and Revision of Representations

Student interview data from both the Whole Number and Fraction Studies provide information on which representations result in mental images and tools that foster students' development of concepts and procedures. In particular, the authors are interested in whether students make connections across

[6] A recent Student Outcomes Study was submitted to the National Science Foundation on student achievement during a Local Systemic Change professional development project in a 100% minority, high poverty district in which the use of *Math Trailblazers* was coupled with a five-year professional development project. During the life of the project, the percentage of students meeting or exceeding state standards at grades 3 and 5 increased significantly from 2000 to 2005. While scores for students at all grades remain significantly lower than scores at the state level, the gap between student outcomes at grades 3 and 5 and corresponding state scores decreased significantly from 2001 to 2005 (Kelso and Zhao, 2006).

representations indicating that they have developed a deeper understanding of these concepts. Teachers' input from the surveys and feedback meetings indicate which contexts and tools are successful in developing the desired understandings. Using the gathered information, authors will carefully review the use of manipulatives and tools at each grade level and make changes as indicated. For example, there is a completed review for grade 1 and number lines are being added to develop concepts of order, addition, and subtraction. Number lines, currently incorporated in a limited way throughout the curriculum, will become a representation that builds concepts systematically from kindergarten to grade 5. The research data suggest that lessons should make more explicit the connections among the various representations in each grade. For example, first-grade lessons will make connections among manipulatives, number lines, number charts, tables, and graphs as well as written and oral representations.

Provide opportunities for good practice

Teachers requested more practice for skill development and more opportunities for students to consolidate concepts. Authors will use information from the completed Classroom Observation Protocols that documents the ways that teachers create these situations in classrooms along with teacher suggestions from surveys to provide activities and explicit suggestions in the materials that will create these opportunities.

Review and Revision of Assessment Tools

In the review of the *Math Trailblazers* Assessment program that was completed last year in collaboration with UIC's Center for the Study of Learning, Instruction, and Teacher Development (LITD), there are calls for 1) clearly laid out observable performance criteria, 2) scoring guides that aid teachers in interpreting students' responses, 3) a developmental framework that can be used to evaluate student progression, 4) modifications to assessments for special needs students, and 5) peer and self-assessment. While the curriculum currently provides many of these tools, they can be improved. In particular, we need to revise the current rubrics included in the curriculum so that they are more teacher and student friendly and so that teachers can adapt them easily to specific assessment tasks and students can use them for self-assessment. Periodically, we will develop a task-specific rubric for assessment activities and provide exemplars of student work.

The revisions process at each grade will begin with a review of the Assessment Indicators that currently serve as a developmental framework. These indicators will serve as a basis for identifying 25 to 30 "big ideas" for each grade and tracking the development of these ideas within grades and across the grades. Using this process, we will revise the Assessment Indicators (and the contents necessary) to make the progression of concepts more explicit. Teacher materials will include more explicit information for teachers on how to use the Assessment Indicators to evaluate and document student progress.

May 2007 Annual Report Insert: Implications for the Revisions Process

Implications for Revision from the Whole Number Study

As curriculum developers, we would like to see most lessons fall into the third row of the Fidelity Grid in Table 5A. That is, we want lesson enactments to have a high level of fidelity to the intended curriculum so that students experience opportunities to reason and communicate as defined in the Classroom Observation Protocol. However, sixteen of the 20 enactments on the grid were coded as having a high level of fidelity to the literal curriculum, but only four enactments were coded as having a high level of fidelity to the intended. Thus, revisions to the curriculum must make the intended curriculum more explicit in the literal curriculum.

The Reasoning and Communication Table (Table 5C) offers insights for revising the curriculum. The revisions team will review lessons within the whole number strand with respect to the progressions in opportunities to reason and communicate on the two axes of the chart. The revisions team will address the following:

- Lessons in grades 2 and 3 that are similar to those that are consistently placed in the top-left quadrant of the table have features that constrain opportunities to learn.

- Lessons that are similar to those that appear in both the top-left and bottom-right quadrants invite multiple interpretations.

- The scarcity of enactments in the bottom-right quadrant implies that few classrooms provide rich student contributions.

Using the examples in the report from the classroom observation team, the revisions team will identify lessons within the whole number strand that are similar to those that fall exclusively in the top-left quadrant of the chart (with only enactments that had a low-level of fidelity to the intended curriculum). These lessons have features that constrain a teacher's implementation by explicitly directing the process students are to follow. These lessons can be modified so that the literal lesson describes more opportunities to reason as defined in the protocol (e.g., select their own representations and strategies [A3] and make comparisons about their peers' representations and strategies [A4]).

In a similar manner, the revisions team will identify and revise lessons that invite multiple interpretations so that the intended curriculum is made more explicit. For example, in the enactments of the *Base-Ten Subtraction* lesson, some teachers implemented the lesson so that students had opportunities to explore the use of the base-ten pieces and develop their own understanding of the subtraction process. In these enactments students developed their own strategies using the base-ten pieces and solved problems using alternative methods and tools. Other teachers taught the lesson by directing students in their use of the base-ten pieces so that students did not have the opportunity to reason or communicate about the procedures. In many of these enactments, the teachers' interpretation of the literal lesson was to use the base-ten pieces to teach the traditional algorithm in a traditional manner. These lessons will be revised to include examples of student strategies and explanations taken from the enactments that were coded as high fidelity to the intended curriculum. Examples of student misconceptions will be included along with support for addressing the misunderstandings.

To address the scarcity of classrooms with rich student contributions, the writers will modify discussion prompts so that they are more open-ended and include a range of sample student responses. They will include information to teachers on why student-to-student conversations are important and sample dialogs that show teachers how to support conversations about mathematics among students. Dialog from the classroom observation tapes serve as examples for new prompts and dialogs. These types of revisions have been included in the first-grade field test materials. Feedback from the grade 1 teachers will be used to inform their use in grades 2 and 3. See Appendix F for examples of discussion prompts and a sample dialog in a lesson from the field test materials in comparison to the use of prompts alone in the same lesson in the current edition.

Analysis of the second-grade student interviews provides evidence that students can and do solve problems using multiple strategies and tools. However, the analysis also indicates that students from classrooms with enactments coded as low-fidelity to the intended curriculum were not as likely as their counterparts in classrooms from high-fidelity enactments to make connections across the representations of their various solutions. Since making such connections is an indicator of conceptual understanding (National Research Council, 2001), the authors will revise lessons so that they provide opportunities to make and explain the connections between representations of numbers and operations. In particular, in grades 2 and 3, students will be asked to compare and explain the similarities and differences in operations represented in multiple ways including the use of number lines, number charts, base-ten pieces, and pencil and paper.

End May 2007 Annual Report Insert: Implications for the Revisions Process

Revisions Process

The revisions process has begun with the following activities:

Review of Assessment Indicators and Representations. The process began with a review of the Assessment Indicators and representations used in first grade as described above. This included an examination of how the content in first grade aligned with state and national standards and with the content in *Math Trailblazers* across the grades. See Appendix E for a table of Assessment Indicators in the Number and Operation Strand.

Development and Discussion of a Concept Paper for Revision of First Grade. The revision team developed and reviewed a concept paper that described a plan for revising grade 1. See Appendix E for an outline of the proposed revisions by unit. These proposals reflect the implications for revision described above and pay particular attention to making connections among representations. The concept paper also outlined a process for revising the units:

1) Author(s) review the following documents and data pertinent to the unit:
   a) Survey data
   b) Consultant reviews
   c) State and national standards
   d) Videos of classroom observations of lessons in the unit along with completed Classroom Observation Protocols
   e) Data from any appropriate student interviews
   f) Sample student work from the unit
   g) Current research literature
2) Assessments are reviewed using LITD protocols for analyzing assessment

3) Authors develop and distribute concept paper for the unit based on the Grade 1 Revisions Framework and above documents
4) Team meets and provides comments and recommendations based on the Concept Paper.
5) Authors write a manuscript for the unit based on comments. Assessments should be developed based on LITD documents
6) Team comments on manuscript and author(s) revise the unit based on comments.

This process is currently under way for units in first grade so that materials will be ready for a field test of grade 1 beginning in September.

February 2007 Update: Revisions Process

The writing and production teams have completed revising the first 11 units of grade 1. All units are being revised for use in the ongoing field test. (Appendix F includes the teacher pages for the current and revised versions of a lesson and samples of Home Practice pages.) The overarching goals of the revisions are to increase students' opportunities to reason and communicate about mathematics as defined in the Classroom Observation protocol. Following the process outlined above, grade 1 units and lessons have been revised in the following ways:

- Most lessons include revised discussion prompts and sample classroom dialogs with examples of student responses. (See pages 65–67 of the Lesson Guide in Appendix F.) When available, transcripts of videotaped lessons provide the basis for prompts and dialogs.
- Each lesson includes a new Summarizing the Lesson section to support teachers and students in discussing the big ideas of the lessons. (See page 68 of the revised Lesson Guide in Appendix F.)
- Flexibility data from the student interviews suggest that most students used multiple tools to solve problems, but very few made connections among the representations. Therefore, the use of existing representations such as ten frames and *100 Charts* has been increased. Activities have been included that make connections among representations explicit. (See pages 65–67 of the revised Lesson Guide in Appendix F for an activity in which students use multiple representations to find all the partitions of ten.)
- The use of number lines has been added to the curriculum in first grade to provide another representation that will be used throughout the curriculum to develop number and operation sense. Each student has a number line from 0–20 on his or her desk, and a class number line from 0–120 is on display. Specific lessons discuss the use of number lines for counting, adding, and subtracting. Other lessons, the Daily Practice and Problems, and the Home Practice encourage students to choose from available tools to solve problems. These tools now include number lines. (See Part 4 of the Home Practice in Appendix F.)
- Survey data indicated that schools and districts required more homework than is currently available in first grade. More homework pages have been added to lessons and four pages of a Home Practice component have been added to each unit. (See the Home Practice in Appendix F.)
- The assessment indicators have been revised for each unit to reflect the revisions in the curriculum. The Assessment Indicators are highlighted at the beginning of each unit and those specific to a given lesson are listed in the Assessment section of each Lesson Guide.
- A new component, Meeting the Needs of All Students, has been included in the Lesson Guides when appropriate. This component provides information to teachers on how to adjust an activity in a lesson to support students who need more attention or those who are ready for a challenge.
- The addition and subtraction strand has been revised to help teachers support students in learning efficient strategies for addition and subtraction. That is, lessons include activities and discussion prompts to help students progress from direct modeling strategies to counting strategies to reasoning from known facts.
- Geometry lessons have been revised based on the consultants' comments.
- Measurement units have been revised to include more activities in which measurement is used as a context for applying counting and computational skills. (See Part 3 of the Home Practice in Appendix F for an example.)

The research team will provide recommendations to the writing team in early March that will inform the revisions for Grade 2. Using this information, a grade 2 writing team will begin its work in the spring. A similar process for Grade 3 will take place during the spring and summer in preparation for a field test of both grades 2 and 3 during the 2007–08 academic year.

Recruitment of Field Test Schools. Forty-seven letters have recently been mailed to schools and districts in an effort to recruit grade 1 classrooms for the field test to begin in the fall. To date, several schools with diverse demographics have responded positively to the letters. An introductory meeting for field test teachers is scheduled for August 14 and 15 on the UIC campus.

May 2007 Annual Report Insert: Revisions Process

The writing and production teams have completed the revision and dissemination of all 19 units and the End-of -Year Test for the grade 1 field test based on the guidelines and procedures in the bulleted list above. Each of the bulleted items will be incorporated into the field test materials for grades 2 and 3. The writing team has completed the following work on the grades 2 and 3 field test materials:

- In a series of meetings, the team reviewed and discussed the survey data on grade 2 unit and lesson usage, teachers' comments from the surveys, the Report on the Grade 2 Classroom Observations, documents summarizing the student interview analysis, the report from the geometry consultant, current national and state standards documents including the NCTM Curriculum Focal Points for grade 2, and a summary of changes to grade 1. Analogous documents for grade 3 are being compiled.
- The current assessment indicators for grade 2 have been organized using the same "big ideas" as in revised assessment indicator documents for the grade 1 field test. This new document provides a framework for reviewing the concepts and skills students are expected to learn throughout second grade by content strand.
- The above information was used to develop a working document that outlines the proposed revisions to the grade 2 units and lessons.
- The writing team developed a more detailed concept paper for Grade 2 Unit 1 and has begun revising lessons.

End May 2007 Annual Report Insert: Revisions Process

February 2007 Update: Grade 1 Field Test

Four hundred students in fourteen classrooms in eight schools in five districts in three states are participating in the Grade 1 Field Test during the 2006–07academic year. In addition to the 14 classroom teachers, three curriculum coordinators and a special education teacher are also participating. The schools include both suburban and urban sites and serve diverse populations, including those that are predominantly White, Hispanic, and African-American. One school includes students who speak over 30 different languages. All participating teachers have previously used *Math Trailblazers*.

For every unit, each teacher receives camera-ready copy of the revised teacher materials and bound copies of the student pages for each student. Teachers complete surveys for each unit similar to the surveys teachers completed in the Whole Number and Implementation Studies. The surveys provide information on which lessons and components teachers use; on which lessons teachers find particularly successful (or unsuccessful) in terms of student learning; on how teachers adapt and supplement lessons, and on teachers' reactions to revisions in the lessons. To date, teachers have submitted a total of 91 surveys via the web.

Participating teachers have attended two field test meetings during the school year, facilitated by members of the research and revision team. A third meeting is planned for June. The field test meetings are intended to provide professional development for participating teachers and an opportunity to provide feedback to the revision team in a discussion format. The professional development activities range from curriculum-based activities to Lesson Reviews, an activity in which teachers analyze videotaped lesson segments.

The overall purpose of the field test study is to provide information to the revision team that will inform the published version of the curriculum and to continue the work of the research team as they examine the use of the curriculum in classrooms. Based on the goals of the revision, the field test study will focus on an examination of the ways external mathematical representations are used and understood by grade 1 teachers and students using the field test materials. The study will pay particular attention to the following external representations embedded within the curriculum, as these representations were included in the revised materials to better support students' understanding of whole number concepts: number lines, ten-frames, part-whole diagrams, and number sentences. The study is guided by the following research questions: (1) how do teachers use and understand particular external representations during instruction, (2) what is the nature of the classroom discourse around particular external representations, and (3) how do students use and understand particular external representations, and (4) to what extent are students able to move flexibly between different representations?

As the research questions state, the study aims to understand classroom processes related to external representations at the teacher, classroom, and student levels. Accordingly, classroom observations, teacher interviews, teacher surveys, student interviews, and student work samples constitute the data sources for the study. The field test teachers were divided into a focus (n = 4) and a non-focus group (n = 11). The purpose of the focus group is to better understand how teachers use and understand particular external representations during instruction to support student understanding and to examine students' understanding of external representation in relation to teachers' instruction. Focus teachers will be observed teaching the same five lessons across the 2006–2007 school year. Pre- and post-observations will be conducted for each teacher for each classroom observation. Within each focus classroom, ten focus students will be chosen at random. Focus students will be interviewed three times during the school year, using a structured interview protocol. The purpose of the interviews is to examine student understanding in relation to certain external representations. Student work samples will be collected from focus students. In the non-focus classrooms, teachers will be observed teaching the same three lessons. Pre- and post-observations will be conducted for each teacher for each classroom observation. Finally, student work samples will also be collected from students in the non-focus classrooms. To date, we have collected six work samples and completed 26 classroom observations and accompanying teacher interviews.

Preliminary feedback from the field test teachers has been thoughtful, both positive and negative, and will be quite useful in the next round of revisions. However, in general, teachers have reacted positively to the overall changes in the lessons and units. In anecdotal reports at the most recent feedback meeting in February, the teachers cited increased student reasoning and communication in their classes this year over previous years.

May 2007 Annual Report Insert: Field Test for Grades 1, 2, and 3

The grade 1 field test is nearing completion. Schools and teachers are being recruited to field test grades 2 and 3 in the 2007-08 academic year. The following bullets summarize progress to date:

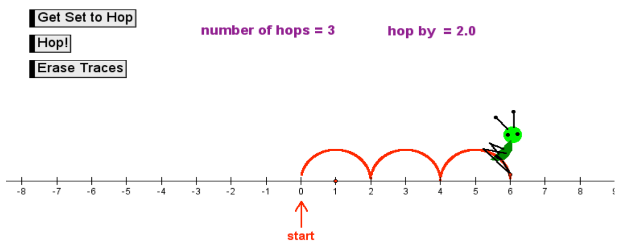
- One hundred sixty-one unit surveys have been submitted electronically by the 18 first-grade field test participants, which include both classroom teachers and curriculum coordinators. The survey data is being compiled so that information from the surveys can inform the second- and third-grade revisions as well as the published edition. In particular, we are currently interested in teachers' reactions to and use of the sample dialogs and revised discussion prompts.
- The research team has completed a total of 40 classroom observations accompanied by teacher interviews. Each of the fourteen classroom teachers has been observed at least once. Student work from each of the observed lessons was collected. The research team has also completed 48 interviews with students from two focus classrooms.
- Thirteen of the 18 grade 1 field test participants will attend a third and final field test meeting on June 18.
- Recruitment letters have been sent to 17 schools including those schools participating in the current grade 1 field test along with additional schools in order to insure similar numbers of classrooms at each grade level and comparable demographic diversity.

End May 2007 Annual Report Insert: Field Test for Grades 1, 2, and 3

Inclusion of Updated Technology

The authors of *Math Trailblazers* are working with KCP Technologies, Key Curriculum Press (the developers of *Geometer's Sketchpad)* and several other curriculum projects to create a pool of computer based mathematics activities for grades 1-8. Some of these activities will be "generic" and marketed directly by Key Curriculum Press, while others will be created or adapted to coordinate with specific curricula, including *Math Trailblazers*. Each activity will consist of a software "sketch" that will run on *Geometer's Sketchpad* plus written teacher materials. Activities will be developed to supplement specific units in grades 1-5. The proposal is to have at least one activity for half the units in grades 2-5. An activity will supplement a specific unit and, in some cases, will provide replacements for certain lessons. Over the past year, KCP Technologies has developed over 40 sketches with accompanying teacher materials. Many of these activities can be adapted to fit with the *Math Trailblazers* curriculum. In addition, we are working directly with developers at KCP Technologies, Rhea Irvine and Daniel Scher. Our immediate goal for this year is to develop ten sketches that we have given the highest priority. To meet this goal, the authors will provide topics and sketch ideas to the KCP developers. There will be at least one sketch for each grade. The first-grade activities will be tested as part of the first-grade field test in the coming year, while others will be tested by a small group of current users of the grade 2-5 *Math Trailblazers* curriculum.

Initially we will work on the following types of activities: computerized versions of the mathhopper lessons in *Math Trailblazers* (jumps on a number line); graphing and making predictions from numerical data that students have gathered (using best fit lines); fraction models (using a circle and rectangular representations of fractions); decimal models (using square grids); flips, slides and turns in the plane (geometry); classification of plane geometric shapes; and investigations involving the relationship between length and perimeter of plane figures.



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Math Trailblazers Research and Revision Project
Appendix A
Research Design and Demographic Data

University of Illinois at Chicago
IMSE

Figure 1. Implementation Study Design

Question: What components of the curriculum do teachers use?

- *What units and lessons do they use? Omit?*
- *How are lessons modified? Supplemented?*
- *What assessments do teachers use?*

Data Collection:

- Written and electronic surveys of curriculum use
- Student work samples
- Pre-observation questionnaires
- Classroom observations
- Post-observation interviews

Question: How do teachers use the curriculum?

- *What are the characteristics of a high fidelity lesson? A low fidelity lesson?*
- *How do teachers extend and enrich the curriculum?*

Data Collection:

- Pre-observation questionnaires
- Classroom observations
- Post-observation interviews

Questions:

- *What revisions to the Math Trailblazers lessons are necessary in order to better support teachers' use of the curriculum?*
- *Which revisions to the Math Trailblazers lessons are necessary in order to better support students' learning of the concepts and operations addressed in the curriculum?*

Approach: Coordinate analysis of the data to better understand how teachers use the curriculum, and the factors that influence teachers' use.

Question: Which factors influence teachers' use of the Math Trailblazers curriculum?

Data Collection:

A. Support

- Interview and survey questions concerning the duration and form of professional development.
- Interview questions concerning on-site support, e.g., common planning time, access to math specialists, pacing guides, etc.

B. Teachers' Perspectives, Orientations and Beliefs

- Surveys and videotaped discussions concerning teachers' beliefs about the weaknesses and strengths of the units and lessons
- Pre-observation questionnaires
- Post-observation interview questions concerning teachers' decisions and classroom practices
- Surveys and videotaped discussions of teachers' evaluations of video segments and characterizations of an ideal lesson implementation.
- Videotaped discussions concerning teachers' evaluations of and characterizations of student work samples.

Figure 2. Whole Number Study Design

Question: What components of the curriculum do teachers use?
- *What units and lessons do they use? Omit?*
- *How are lessons modified? Supplemented?*

Data Collection:
- Written surveys
- Student work samples
- Pre-observation questionnaires
- Classroom observations
- Post-observation interviews

Question: How do teachers use the curriculum?
- *How are whole number lessons enacted in classrooms?*
- *What are the characteristics of a high-fidelity lesson?*
- *What are the characteristics of a low-fidelity lesson?*
- *How do teachers expand and enrich the curriculum?*

Data Collection:
- Pre-observation questionnaires
- Classroom observations
- Post-observation interviews

Questions:
- *Which revisions to the Math Trailblazers whole number lessons are necessary in order to better support students' learning of whole number concepts and operations?*
- *What revisions to the Math Trailblazers whole number lessons are necessary in order to better support teachers' use of the curriculum?*

Approach: Coordinate analysis of the data to better understand how teachers use the curriculum and the factors that influence teachers' use.

Questions:
(1) To what extent are Math Trailblazers' students developing the whole number concepts and operations that are the foci of the Math Trailblazers whole number strand.
(2) How are students' understandings of whole number concepts and operations related to their experiences with the Math Trailblazers whole number lessons?

Data Collection:
- Classroom observations
- Periodic student interviews coordinated with classroom observations
- Student work samples coordinated with classroom observations
- Additional student work samples
- Student achievement data

Question: Which factors influence teachers' use of the Math Trailblazers whole number lessons?

Data Collection:

A. Support
- Interview and survey questions concerning the duration and form of professional development.
- Interview questions concerning on-site support, e.g., common planning time, access to math specialists, pacing guides, etc.

B. Teachers' Perspectives, Orientations and Beliefs
- Surveys and videotaped discussions concerning teachers' beliefs about the weaknesses and strengths of the units and lessons
- Pre-observation questionnaires
- Post-observation interview questions concerning teachers' decisions and classroom practices
- Surveys and videotaped discussions concerning teachers' evaluations of video segments and characterizations of an ideal implementation.
- Videotaped discussions concerning teachers' evaluations and characterizations of student work.

Demographic Information

Table 1. Demographic Information – Implementation Study

School	# of Classrooms	Grades	Predominant Ethnicity	Location	Income	Length of Use
A	3	1, 3	White	Suburban	Middle-High	Short
B	6	K, 4	White	Suburban	Middle-High	Short
C	7	K–2	White	Rural	Low-Middle	Intermediate
D	2	K, 2	African Am.	Suburban	Low-Middle	Intermediate
E	4	K, 2, 3, Sp. Ed.	Hispanic & White	Urban Large City	Low-Middle	Short
F	2	5	White	Suburban	Middle-High	Short
G	3	4, 5	African Am.	Urban Large City	Low	Short
H	4	3–5	Am. Indian	Rural	Middle	Short
I	2	4	Hispanic & African Am.	Urban Large City	Low-Middle	Intermediate
J	1	3	White & African Am.	Suburban	Middle	Intermediate
K	1	5	Hispanic	Urban Large City	Low	Short
L	7	1, 2	White & African Am.	Suburban	Middle	Long
TOTAL	**42**					

Table 2. Demographic Information – Whole Number Study

School	# of Classrooms	Grades	Predominant Ethnicity	Location	Income	Length of Use
L	2	3, 5	White & African Am.	Suburban	Middle	Long
M	4	K, 2, 4	White	Suburban	Middle-High	Intermediate
N	1	K	White	Suburban	Middle-High	Intermediate
O	11	K–4	White	Urban Midsize City	Middle	Short
P	8	1–5	Hispanic	Urban Large City	Low-Middle	Intermediate
Q	8	1–5	Mixed	Urban Large City	Low-Middle	Long
R	6	K, 2, 3, 5	African Am.	Urban Large City	Low-Middle	Long
S	1	1	African Am.	Urban Large City	Low	Long
T	4	3, 5	Mixed	Urban Large City	Low	Short
TOTAL	**45**					

Math Trailblazers Research and Revision Project
Appendix B
Classroom Observation Protocol Documents

University of Illinois at Chicago
IMSE

DEFINITIONS

OPPORTUNITY TO LEARN Definition: An opportunity to learn during an enacted lesson occurs when a question posed by a teacher, within a curricular activity, or during small group or whole class discussions creates a context by which students may expand or modify their existing conceptions, procedures, or classroom practices.
A. Opportunities to Reason
(A1) Reason to solve problems; Reason about a mathematical concept. Definition: Situations in which students may either explore how to use a novel tool, representation, or strategy while solving familiar problems, or apply familiar tools, representations, or strategies to novel problems. Situations in which students describe and/or consider mathematical attributes or properties. For example, while describing what they see in a list of number sentences, students may observe that 20 + 80 = 100 and 80 + 20 = 100 “are the same” because in both cases they are “putting together the same parts.” In so doing, the students are considering the commutative property of addition.
(A2) Use or apply concepts, strategies, or operations; refine strategies so that they become more efficient. Definition: Situations in which students may refine their application of concepts, strategies, or operations; or develop their repertoire of strategies or operations by applying them to problem situations.
(A3) Select from multiple tools, representations, or strategies. Definition: Situations in which students may consider a variety of tools, representational approaches, or strategies in an effort to make appropriate choices based on problem situations. This category includes situations in which students spontaneously select and include tools while problem solving.
(A4) Compare and make connections across tools, representations, or strategies. Definition: Situations in which students may make connections by moving between representations or strategies, or by discussing differences, similarities, and ways of using various representations or strategies.
(A5) Validate strategies or solutions; reason from errors; inquire into the reasonableness of a solution. Definition: Situations in which students may evaluate the logic of strategies, or the reasonableness and accuracy of solutions. Situations in which students may use errors as a basis for further investigation.

B. Opportunities to Communicate
(B1) Communicate mathematical ideas or ways of reasoning. Definition: Situations in which students may describe mathematical ideas (e.g., patterns or conceptions of sameness) or describe their use of tools, representations, or strategies to peers, the teacher, or in writing.
(B2) Interpret another student’s way of reasoning about tools, representations, strategies, or operations. Definition: Situations in which students may respond to, explain, or question another student’s approach to a problem.
(B3) Clarify or justify reasoning or explanations. Definition: Situations in which students may refine their explanations making them clearer and more complete. Situations in which students may provide arguments that support their reasoning or solutions.
(B4) Characterize mathematical operations. Definition: Situations in which students may either describe an operation as it applies to a group of problem situations or generalize across problem situations to answer questions similar to “what are we doing when we trade (or some other operation)?” For example, students may describe partitive division as equal sharing among groups as they describe what they are doing when they divide. Another example would be when students describe the concept of addition as finding how many they have altogether.

BLANK PROTOCOL

Protocol Status:	
Date Completed:	
Evaluator:	

Demographic Information	
Study	
Grade	
Teacher Code	
School Code	
Edition	
Length of Use (School)	
Length of Use (Teacher)	
Amount of Professional Development	
Observation Date	
Lesson Observed	

Site Information	
School Size	
Ethnicity	
Income	
School Location	

About the Lesson
Prior lessons and their relation to the lesson
Lesson Guide Recommendations to Teachers What recommendations help teachers get at the key content (might be tasks, but also including listing questions that are mathematical questions that help teachers get to the mathematics Set up: Procedure: Discussion points or suggested questions:
Future lessons and their relation to the lesson
Mathematical focus of the lesson

Literal Lesson Evaluation
[either *Implemented*, *Partially Implemented*, or *Not Implemented*]

Set up	**Implementation**
Procedure	**Implementation**
1.	
2.	
3.	
4.	
Discussion Points or Suggested Questions	**Implementation**

Observation Evaluation	
1. Materials Available Please indicate which materials were observed during the lesson.	
(a) base-ten pieces	
(b) 100 \ 200 chart	
(c) unifix cubes	
(d) links	
(e) other	
2. Use of the Curricular Unit/Lesson/Activity Please describe any modifications to the curriculum, including but not limited to:	
(a) Prepared handouts not included in the curriculum.	
(b) Curricular items removed from the lesson.	
(c) Items included in the enacted lesson that are not part of the curriculum.	
(d) Teacher's extensions/modifications via posed tasks/questions during the lesson	
(e) Other	

3. Opportunities to Learn—Codes

Lesson Specifications		**Mathematical Foci**							
(A) Opportunities to explore, use, and deepen mathematical knowledge.		1	2	3	4	5	6	7	8
(1) Reason to solve problems; Reason about a mathematical concept.									
(2) Use or apply concepts, strategies, or operations; refine strategies so that they become more efficient.									
(3) Select from multiple tools, representations, or strategies.									
(4) Compare and make connections across tools, representations, or strategies.									
(5) Validate strategies or solutions; reason from errors; inquire into the reasonableness of a solution.									
(B) Opportunities to communicate about mathematics.		1	2	3	4	5	6	7	8
(1) Describe ways of reasoning about tools, representations, strategies, or operations.									
(2) Clarify or justify reasoning or explanations.									
(3) Interpret another student's way of reasoning about tools, representations, strategies, or operations.									
(4) Characterize mathematical operations.									

Summary

Issues Related to the Lesson and its Enactment

Enactment Categorization
Teaching Style Code: Explanation: Community Type Code: Explanation: Level of Fidelity Code: Explanation:

Additional Comments

BLANK FIDELITY GRID

Level of Fidelity to Intended and Literal and Curricula (Intended, Literal)			
Literal / Intended	**Low: Most of the lesson guide recommendations were not implemented**	**Moderate: Though many lesson guide recommendations were followed, key recommendations were not implemented**	**High: Most of the lesson guide recommendations were implemented**
Low: Most opportunities to learn in the enacted lesson fail to align with intended curriculum	(low, **low**)	(low, **moderate**)	(low, **high**)
Moderate: Though many of the opportunities to learn aligned with the intended curriculum, some key opportunities to learn were also missed during the enacted lesson.	(moderate, **low**)	(moderate, **moderate**)	(moderate, **high**)
High: Most of the opportunities to learn in the enacted lesson align with intended curriculum	(high, **low**)	(high, **moderate**)	(high, **high**)

SAMPLE PROTOCOL

Protocol Status:
Date Completed:
Evaluator:

Demographic Information	
Study	
Grade	
Teacher Code	
School Code	
Edition	
Length of Use (School)	
Length of Use (Teacher)	
Amount of Professional Development	
Observation Date	
Lesson Observed	

Site Information	
School Size	
Ethnicity	
Income	
School Location	

About the Lesson
Mathematical focus of the lesson
1. Represent multiples of ten and one hundred using links and number sentences. 2. Group and count objects by tens. 3. Partition one hundred into groups of ten. 4. Explore the relationship between addition facts for ten and multiples of ten. 5. Write number sentences for addition situations. 6. Solve addition and subtraction problems using multiples of tens. 7. Develop ten as a composite unit. 8. Explore part-whole relationships. The primary focus of *100 Links* is the development of 100 in terms of different referent units via the partitioning of 100 into two parts which are then described in terms of groups of tens and groups of links. In so doing the students should move towards being about to think flexibly about the quantity 100 as 1 hundred (represented with one 100-link chain), 10 tens (represented with ten 10-link chains), and 100 ones (represented with 100 links). Secondary to the development of 100 in terms of different referent units is the development of connections between basic addition facts for ten and for multiples of ten. This lesson should also support students development of ten as composite unit (i.e., ten simultaneously conceived of as one unit of ten and ten units of one) as the students move between descriptions of the parts as groups of ten (e.g., two groups of ten plus eight groups of ten equals ten groups of ten) and as a number of links (e.g., twenty links plus eighty links equals one hundred links).

Prior lessons and their relation to the lesson.

1. Exploring representations of numbers:

Unit 3	(L2) Use the ten frame to visualize numbers with 5 and 10 as benchmarks.
Unit 5	(L2) Use the ten frame to form and count groups of 10 pennies.
Unit 9	(L4) Find target numbers using number relationships on the 100s chart.
	(L5) Count by 2s, 5,s 10s and identify target numbers using patterns and relationships between numbers on the 100 chart.
	(L6) Measure lengths of objects using links. The links are set up in groups of 10 of the same color.

2. Partitioning numbers:

Unit 3	(L6) Partitioning 10 by placing pennies in two groups
Unit 4	(L2) Partition 11 into two and three groups using counters
	(L4) Explore partitioning numbers using "Counting-on Cards"
Unit 8	(L4) Partition a group of 10 beans using a ten frame.

3. Exploring part-whole relationships:

Unit 3	(L5) Explore part-whole relationships by counting pockets.
Unit 4	(L2) Solve part-whole problems with animals in a pet store.
Unit 8	(L1) Solve part-whole problems with circus animals.
	(L2) Use a part-whole diagram to model part-whole number stories.

…continued for each focus statement

Future lessons and their relation to the lesson.

Exploring representations of numbers; forming groups of 10 and counting by 10s; and exploring the relationship between groups of ten and basic facts:

Unit 11	(L2) Compare dimes to rows on hundred chart and to groups of 10 links in 100-link chain. Focus on the relationship between basic facts and extensions of basic facts (with groups of ten).
	(L3) Count by 5s and 10s and find coin combinations with groups of 5s and 10s that make a $1.00.
	(L4) Navigate by 10s on the 100 chart.
Unit 12	Grouping and counting.
Unit 17	Represent numbers greater than 100; group and count by 10s and 100s; solve addition problems using multiples of 10s and 100s

Partitioning numbers and writing number sentences:

Unit 13	Partitioning 10; writing number sentences

Lesson Guide Recommendations to Teachers What recommendations help teachers get at the key content (might be tasks, but also including listing questions that are mathematical questions that help teachers get to the mathematics)

Set up:
Students work in groups of 4. Each group receives 50 links of one color and 50 of a second color. Students sort their links into same-color groups of 10.

Procedure:
1. Students make a 100-link chain and compare the lengths of their chains.
2. The class discusses how many ten-links chains make up the 100-link chain. There is a TIP to: relate 10 rows on hundred chart to 10 ten-link pieces of 100-link chain.
3. Students break their chains into two parts and record an addition number sentence representing the partition and the total.
4. Students share how they broke their chains by showing both parts and telling how many groups of 10 they have in each part. They review how many groups of 10 there are in the entire chain. Then translate groups of ten into the number of links in each part of the chain. Finally they record their number sentences on the board.
5. Teacher prompts the class to add any "missing" number sentences to the list on the board. The teacher

asks the students to describe the number of groups of ten as well as the number of individual links. 6. Discuss any number sentences that have reversed addends, and explore whether or not students think the number sentences are the same or different. 7. Use the questions provided to conclude the first part of the lesson (partitions into two parts). 8. Students work on finding as many 3-partition number sentences as possible for their 100-link chains. Discussion points or suggested questions: • When students partition their chains, have them compare the number of groups of ten to the number of individual links for each section of the 100-link chain. The intention is to explore the relationship between addition facts for ten and multiples of tens.

Literal Lesson Evaluation
[either *Implemented*, *Partially Implemented*, or *Not Implemented*]

Set up	**Implementation**
Students work in groups of 4. Each group receives 50 links of one color and 50 of a second color. Students sort their links into same-color groups of 10.	Implemented
Procedure	**Implementation**
5. Students make a 100-link chain and compare the lengths of their chains.	Partially Implemented Students make their 100-link chains as described in the lesson, but they do not compare the lengths of their chains.
6. The class discusses how many ten-links chains make up the 100-link chain. There is a TIP to: relate 10 rows on hundred chart to 10 ten-link pieces of 100-link chain.	Implemented
7. Students break their chains into two parts and record an addition number sentence representing the partition and the total.	Implemented
8. Students share how they broke their chains by showing both parts and telling how many groups of 10 they have in each part. They review how many groups of 10 there are in the entire chain. Then translate groups of ten into the number of links in each part of the chain. Finally they record their number sentences on the board.	Partially Implemented Students do share how they have broken the hundred-link chain in different ways, but because the teacher modified the student page to include four number sentences, students did not necessarily have their chains partitioned like the number sentence that they added to the class list. Students did not hold up their chains to show the addends for the partitions.
9. Teacher prompts the class to add any "missing" number sentences to the list on the board. The teacher asks the students to describe the number of groups of ten as well as the number of individual links.	Implemented
10. Discuss any number sentences that have reversed addends, and explore whether or not students think the number sentences are the same or different.	Implemented (They did not discuss if they were the "same," but the discussion about these did highlight the relationship between the number sentences.)
11. Use the questions provided to conclude the first part of the lesson (partitions into two parts).	Implemented
12. Students work on finding as many 3-partition number sentences as possible for their 100-link chains.	Implemented

Literal Lesson Evaluation (continued)

Discussion Points or Suggested Questions	Implementation
When students partition their chains, have them compare the number of groups of ten to the number of individual links for each section of the 100-link chain. The intention is to explore the relationship between addition facts for ten and multiples of tens.	Implemented

LEVEL OF FIDELITY TO THE LITERAL LESSON	HIGH

Observation Evaluation	
I. Materials Available Please indicate which materials were observed during the lesson.	
(A) base-ten pieces	
(B) 100 \ 200 chart	100-chart is referred to in the introduction
(c) unifix cubes	
(d) links	Making 100-link chains
(e) other	
II. Use of the Curricular Unit/Lesson/Activity Please describe any modifications to the curriculum, including but not limited to:	
(A) Prepared handouts not included in the curriculum.	Teacher provided a page that had four two-part addition sentence problems before passing out the student page that is provided with the lesson.
(b) Curricular items removed from the lesson.	It did not appear that students compared their hundred-link chains to see that they were the same length.
(c) Items included in the enacted lesson that are not part of the curriculum.	Teacher had fifty-link chains of one color prepared ahead of time and invited students to use these to make a hundred-link chain with alternating colors.
(d) Teacher's extensions/modifications via posed tasks/questions during the lesson	Teacher asked students for the missing addend Teacher asked students to think about how they can check their work.
(e) Other	N/A

3. Opportunities to Learn—Codes

Lesson Specifications		**Mathematical Foci**							
(A) Opportunities to explore, use, and deepen mathematical knowledge.		1	2	3	4	5	6	7	8
(1) Reason to solve problems; Reason about a mathematical concept.	[00:07:30.07–00:08:25.00] Small Group Students reason about how to use their links to represent one hundred in groups of ten and how to build their hundred-link chains by connecting alternate colors of ten-link chains to make it easier to count by tens.		X					X	
	[00:19:46.16–00:20:19.29] Whole Class Students reason about how to record a number sentence to represent the partitioned hundred-link chain. They compare the addends in basic facts (representing the number of groups of ten) with the addends in their number sentence (representing the total number of links in each piece of the chain).				X	X			
(A) Opportunities to explore, use, and deepen mathematical knowledge.		1	2	3	4	5	6	7	8
(2) Use or apply concepts, strategies, or operations; refine strategies so that they become more efficient.	[00:22:44.17–00:26:11.28] Small Group Students reason about how the partitioned hundred-link chain can be represented in a number sentence as they partition their hundred-link chains into two pieces and record a number sentence to represent their work. See—Male Student at [00:23:00.04] and Male Student at [00:24:30.08] and Female Student at [00:25:21.16] and Sophia at [00:25:43.25].			X		X			
	[00:38:54.21–00:43:23.10] Small Group ADDITIONAL CONTENT: Explore using the associative property to find missing addends and to check work. Students repeatedly partition their chains into three parts and record addition number sentences to represent their work. The teacher invites them to find different ways of partitioning the chain.			X		X			X

Lesson Specifications		Mathematical Foci							
(A) Opportunities to explore, use, and deepen mathematical knowledge.		1	2	3	4	5	6	7	8
(3) Select from multiple tools, representations, or strategies.	NOT APPLICABLE TO LINKS LESSON								
(A) Opportunities to explore, use, and deepen mathematical knowledge.		1	2	3	4	5	6	7	8
(4) Compare and make connections across tools, representations, or strategies.	[00:15:05.05–00:17:57.08] Whole Class ADDITIONAL CONTENT: Solve missing addends problems with multiples of tens. Teacher uses the chain to model missing addend problems for students. Students have the opportunity to connect the concrete partitioning of the chain to finding the missing addends in a number sentence.			X		X	X		X
	[00:36:01.24–00:37:39.19] Whole Class ADDITIONAL CONTENT: Solve missing addend problems with multiples of ten. LIMITED OPPORTUNITY—The teacher records a number sentence based on the pieces of hundred-link chain she holds, helping students connect the addends to the pieces of chain. The teacher does not directly connect the addition facts she is asking the students to solve with the multiples of ten in the partitioned hundred links, although it is clearly her goal. The students who are answering appear to be making this connection, but others seem to be confused.			X	X	X	X	X	X
(A) Opportunities to explore, use, and deepen mathematical knowledge.		1	2	3	4	5	6	7	8
(5) Validate strategies or solutions; reason from errors; inquire into the reasonableness of a solution.	[00:12:30.25–00:14:02.12] Whole Class POSSIBLE MISSED OPPORTUNITY—Students gave incorrect answers. The teacher did not ask them to explain their thinking or how they got their answers, or to judge the reasonableness of their answers. The teacher waited until someone gave the correct answer and accepted it. Without asking students to think about the correct answer, she had them verify it by counting.						X	X	X
	[00:15:05.05–00:17:57.08] Whole Class ADDITIONAL CONTENT: Solve missing addends problems with multiples of tens. The teacher has students model checking the addends in number sentences by counting the number of links in partitions of the hundred-link chain.			X		X	X	X	X

Lesson Specifications		Mathematical Foci							
(B) Opportunities to communicate about mathematics. Please describe opportunities available to the students to		1	2	3	4	5	6	7	8
(1) Describe ways of reasoning about tools, representations, strategies, or operations.	[00:15:05.05–00:17:57.08] Whole Class ADDITIONAL CONTENT: Solve missing addends problems with multiples of tens. The teacher asks students to explain their strategies for finding the number of links in the second part of the hundred-link chain. See—Female Students at [00:15:38.20], [00:17:08.26], and [00:17:21.13]			X	X	X	X	X	X
	[00:38:54.21–00:43:23.10] Small Group ADDITIONAL CONTENT: Explore using the associative property to find missing addends and to check work. There is some discussion about the total links in each partition. Students explain the relationships between the multiples of ten they are using and the total. They also use the associative property to check their work. See discussions at: [00:38:54.21] and [00:39:30.11] and [00:40:04.18] and [00:40:33.20] and [00:42:24.19].			X		X			X
(B) Opportunities to communicate about mathematics. Please describe opportunities available to the students to		1	2	3	4	5	6	7	8
(2) Clarify or justify reasoning or explanations.	[00:18:49.27-00:21:41.01] Whole Class POSSIBLE MISSED OPPORTUNITY—The teacher does not ask students to elaborate on their response of “we counted.” It is unclear if they counted the links by tens or by ten-link sections, or if they found the missing addends by counting on from the first addend.	X				X		X	X
	[00:39:15.00-00:41:03.10] Small Group Students share their answers. The teacher asks one student to model how he solved the problem by counting. He demonstrates that there is a mistake by counting on his fingers to show that they need ten units of ten and therefore the missing addend when they already have two sections of three is four.			X	X				X

Lesson Specifications		Mathematical Foci							
(B) Opportunities to communicate about mathematics. Please describe opportunities available to the students to		1	2	3	4	5	6	7	8
(3) Interpret another student's way of reasoning about tools, representations, strategies, or operations.	[00:17:57.08–00:18:38.04] Whole Class ADDITIONAL CONTENT: Explore the relationship between the operations of addition and subtraction. LIMITED OPPORTUNITY—It appears that a male student might be building off of a classmate's thinking when, after his classmate suggests subtracting to find the answer, the male student suggests working backward to find a missing addend . The teacher does not ask the male student to clarify whether he is making this connection, but dismisses his suggestion by stating that today's lesson is about addition.						X		
	[00:18:17.21-00:18:49.27] Small Group One student clarifies another classmate's thinking about whether the answer is twenty or thirty. He shows two fingers for the two ten-link sections in the partition, and says that is twenty. He holds up a third finger to show how many sections of ten-link chain would be in thirty.			X	X				X
(B) Opportunities to communicate about mathematics. Please describe opportunities available to the students to		1	2	3	4	5	6	7	8
(4) Characterize mathematical operations	This category may not be applicable in Grade 1.								

LEVEL OF FIDELITY TO THE INTENDED LESSON	MODERATE

Summary

Issues Related to the Lesson and its Enactment

Teacher does not make her mathematical focus clear. It is therefore difficult to tell what her goals are for the lesson activities and discussions. It is also not clear that students know what mathematics they should be thinking about.

Enactment Categorization

Teaching Style
Code: TO BE DETERMINED

Community Type
Code: TO BE DETERMINED

Level of Fidelity
Code: TO BE DETERMINED

Additional Comments

It was not clear whether students made connections as they progressed through the lesson. The teacher seemed to emphasize the connections between basic facts and groups of ten, and there were instances where the students applied this skill. Some students did seem to connect the partitions with the number sentences, but it was difficult to tell from the video how prevalent this was. There were few real conversations, and little exploration on the part of the students.
Although the teacher did follow the recommendations in the URG, the more general suggested practices for MTB were ignored—e.g., asking probing questions, have the students engage each other in discussing, questioning, exploring, clarifying, etc. It does appear that students were able to partition the hundred-link chain and record number sentences correctly, but due to some incorrect answers (that were never addressed by the teacher), it is not clear how much or how deeply the students understand.

Math Trailblazers Research and Revision Project
Appendix C
Whole Number Study Student Interview Analysis Documents

University of Illinois at Chicago
IMSE

RUBRICS FOR SCORING STUDENT INTERVIEWS

	0	1	2	3
Accuracy	**Almost all incorrect responses**. Does not have 1-1correspondence, so gets different results when counting (but possibly can rote count by ones to ten or tens to 100). For the graphing problem, when asked how many children had cats (6) or fish (7), student responded 67. Demonstrated using square tiles, with 6 in one column and 7 in the other. When asked how many, she made the columns closer to indicate 67.	**Two or more incorrect responses.** Student incorrectly counts on to get a sum that is off by one. For the links task, when determining the number of links the student is off by ten. For the partition of 60 and 40, a student counts his 60 and says the interviewer has 30.	**One incorrect response, possibly two if many questions were asked.**	**Responds correctly to all tasks.**

	0	1	2	3
Reasoning Level	**No meaningful strategy.** Does not focus on the meaning of the word problem. Needs explicit directions.	**The strategy demonstrates that the student understands the operation and must directly model the problem to solve it.** S employs a count all strategy. For example, for 11+12=23, S counts out 11 cubes, another set of 12 cubes and then counts the collection. S is able to count by tens, however student seems to have difficulty with the base ten system and/or the representations she is using. For example, when counting a three-link chain the student counts by tens. When working the problem of 30 plus 60 student replies ¯I remember how to say it...60 hundred and 30 hundred.	**The strategy demonstrates that the student understands the operation and can apply counting to solve the problem.** I: Suppose that I go home and I boil 9 eggs, and none crack, and you go home and boil 12 eggs, and none crack. How many do we have together? S: Picks up pen. Writes something and goes to cubes. Counts out by twos to 9 and counts out 12 by ones adds by twos. hum, 21 I: 21? S: Shakes head yes I: So how many eggs do we have all together? Are you pretty sure of that? Don't want to check it? S: Shakes head yes	**The strategy demonstrates that the student is able to apply mathematical reasoning about relations between numbers to accurately solve the problem.** Compares a ten-link with another ten-link to show that they are both the same length. Knows that 4+6=10, so 40+60=100. 28+5 is more than 30 because 8+5 is more than ten. "I remembered that 11 + 11 is 22 and one more is 23." "I knew there were 20 in there 'cause I took out 3 (from the 23). "Because I know that 50+50=100 and 5+5=10."

	0	1	2	3
Flexibility with tools or representations	**Not able to use a tool or representation to solve problems.** .	**Can use one tool or representation to model mathematical concepts and solve problems.** Able to offer another tool or representation but has difficulty using it. For example, S used fingers primarily to solve the task. S gives one possibility of doing a problem another way. When asked to do 12+9=21 another way, student replies 9+12=21. Otherwise student is unable to give another way to solve the task given to him.	**Can use multiple tools or representations to model mathematical concepts and solve problems, but might not make connections across the representations.** I: How many do we have all together? S: [was writing as interviewer asked question] 11+ 12 writes the numbers and goes to the 100's chart then writes 23 I: Can you explain to me what you did? You did a nice job. S: I started at 12 and counted up 11. I: And you counted by ones? S: Yes	**Can use multiple tools or representations to model mathematical concepts and solve problems.** **Recognizes connections among representations.** Knows 7+3=10 "from the clock." Knows 34+10=44 from dimes, i.e., "three dimes and one dime is forty, plus 4, is 44."

	0	1	2	3
Communication	**Explanation and/or description totally unclear or irrelevant; supporting argument not present; use of pictures, symbols, tables and graphs are not present or completely inappropriate; does not use appropriate terminology.** No descriptions or explanations given, even when asked.	**Explanation and or description is possibly ambiguous or unclear (minimal); supporting arguments are incomplete or logically unsound; use of pictures, symbols, tables and graphs are present, but with errors and/or irrelevant; terminology used with major errors.** S Gives vague explanations, like she used “fingers,” or ¯did it in her head.”	**Explanation and or description is fairly complete and clear; supporting arguments are logically sound, but may contain minor gaps; use of pictures, symbols, tables and graphs are present, but with minor errors and/or somewhat irrelevant; terminology used with minor errors.** S: Counted back 8…7 I: You counted back 7? Can you do that out loud for me? S: 15,14, 13, ... using his finger to count back 7.	**Explanation and or description is complete and clear; supporting arguments are strong and sound; use of pictures, symbols, tables and graphs are correct and clearly relevant; terminology is clear, precise and appropriate.**

SAMPLE STUDENT PROFILES

Student #: 103P--Female

Links task: 5 questions were asked out of 10: Student was not asked questions #4-8.
Graph task: 10 questions where asked out of 12

ACCURACY

Links task: 4/5 (correct/ total # asked)
Graph task: 10/10(correct/ total # asked)

REASONING LEVEL

Links task: Level 1 for problem 1, which indicates that the student has one-to-correspondence. Level 2 overall for the other questions. Student is able to use knowledge of doubles however primarily uses a counting by tens strategy to find both parts of the link chain.
Graph task: Level 2 overall. Student uses fingers to solve problems by counting on.

STRATEGIES USED, MODELS, AND NUMBER SENTENCES

Links task:

(1) Student counts link by ones up to 10

(2a) [30 + 70] Counts the first part by tens to 30. Counts the second part by tens to 70. <u>30 + 10 + 10 + 10 + 10 + 10 + 10 + 10</u>. Writes “30 + 70 =100” and “70 + 30 = 100.”

(2b) Counts the first part by tens to 50. Understands that the second part is 50 because “<u>50 + 50 = 100</u>.” Writes 50 + 50 =100.

(3a) [30 + 70] Counts the first part by tens to 30. Understands that the second part is 70 because of recent experience with the same problem.

(3b) [20 + 80] Visually recognizes 20. Counts up 10 from 80 twice to 100.

Graphs task:

(1) Strategy 1: (Fingers) Used fingers to model problem, puts eight fingers up and then counts on: one, two, three, four, five, six. Answers fourteen.
Strategy 2: (Numbers chart) Using 100s chart points to eight and counts up…one, two, three, four, five, six. Student writes 8+6=14 cats and fish for a number sentence.

(2a) Fact retrieval. Student draws a line with her finger across the graph and replies eighteen. Student explains that she knows eight and six is fourteen, and adds four more is eighteen mentally. Student write the number sentence 8+6+4=18.

(2c) Fact retrieval. Student replies that cats, fish and birds has more since eighteen is bigger than twelve.

(3) Fingers: Student uses a counting up strategy from eight to figure out how many more dogs than fish. Writes 12– 8=4. When explaining what the numbers in the number sentence mean student replies “*um, twelve, there's twelve dogs and then there is eight fish and so it...so then you take away, um, and then you take away eight and it has four fish, four more to make twelve. For the fish to get up, to go up.*”

(5) 100s Chart. Student makes up the problem “ dog plus cat plus fish plus turtle plus birds” writing 12+6+8+2+4= . Solves using the number chart. Explains that you should start with the biggest number, then go to the lowest number. Solves the problem correcting using this strategy.

CONNECTIONS BETWEEN MANIPULATIVES

Graph task:

(2a) Connection between the number sentences for fish, cat and birds (8+6+4=18) and fish and cat and fish (8 +6=14). Uses the fact that 8+4= 14 to get that 8+6+4=18.

CONTENT INDICATORS	YES	NO	NO EVIDENCE
1. Can student count objects by tens? (Links)	X		
2. Can student solve addition and/or subtraction problems involving multiples of ten? (Links)	X		
3. Does student make connections between basic addition facts for ten and multiples of ten? (Links)			X
4. Can the student count on to solve addition problem situations? (Cubes)			X
5. Can a student count on or back to solve a subtraction situation? (Cubes)			X
6. Can the student read data from bar graphs? (Graph)		X	
7. Can the student use data to solve problems? (Graph)	X		
8. Can the student represent numbers using manipulatives and_number sentences? (Links, Graph, Cubes)	X		

Student #: 104P---Male

Links task: 6 questions were asked out of 10: Not asked 5-8
Graph task: 11 questions where asked out of 12

ACCURACY

Links task: 6/6 (correct/ total # asked)
Graph task: 11/11: Student obtained the wrong information from the graph on number 2 (was off by 1) for the number of fish, but solves the addition problem with his numbers correctly.

REASONING LEVEL

Links task: Level 1 for problem 1 which indicates that the student has one-to-correspondence.
Level 3 for all other tasks, which indicates that student was able to model the situation by using the number facts for sums to 10.
Graph task: Level 3 in general (however continues to use some counting strategies).

STRATEGIES USED, MODELS, AND NUMBER SENTENCES

Links task:

(1) Student counts first group of tens by one; counts second group of tens by twos

(2a) [20 + 80] Visually recognizes that two groups of 10 is 20. Counts the other part in groups: 30 + 10 + 10 + 10 and recognizes that the other two groups make 20. <u>30 + 10 + 10 + 10 + 20 = 80.</u>

(2b) [60 + 40] Counts the first part by 10s to 60. Understands that the second part is 40 because "<u>60 + 40 = 100</u>." Writes 60 + 40 = 100.

(3a) [50 + 50] Counts the first part by 10s to 50. Understands that the second part is 50 because "50 + 50 = 100" and "<u>5 + 5 = 10.</u>" Writes 50 + 50 =100.

(3b) [70 + 30] Visually recognizes the first part as 70. Understands that the second part is 30 because "30 + 70 = 100" and "<u>7 + 3 = 10</u>."

Graph task:

(1) Strategy 1: Doubles. Used doubles to find (1+(6 + 6)= (1+6) +6 [writes 7+6=13]

Strategy 2: Graph. Describes a method using the graph of adding on six to the fish bar since it has more than cat bar.

(2a) Mental computation. Uses a mental math strategy of (5+6) – 1 +8 + 1. So that he can use an addition fact involving ten.

um, there's five plus six equals eleven, and then um, just like if this, just like if this was five then i would just add on eight more then it would be eighteen, but since um, but since, but since um bird and um cat equal eleven i just have to add one more so it will equal nineteen.[writes5+6+8=19]

(2c) Graph. Uses the graph to decide that graph of bird, fish and cat would go over dog bar.

(3a) Graph. Model 8 + ? = 12. Uses the graph to count up from the fish bar (which he now reads as 8 instead of 7) and looked across to the dog bar to see when the heights matched. [writes 8+5=12]

(4) Skip Counting. Counted by fives to get fifteen. [writes 5+5+5=15]

(5) Writes 4+3=7 to explain:

there's four birds and i just counted up until i got to fish so it would be um four plus three equals seven because there's seven fish.

CONNECTIONS BETWEEN MANIPULATIVES

Links task:

(2a) After writing number sentence, "20 + 80 = 100," student represents the problem using knowledge of fact family to produce, "100 – 20 = 80."

I: Can you write that for me in a number sentence?

S: [writes 2 and vertical line and then inserts 0 between the 2.. Crosses the vertical line to make + sign and then writes 80 = 100. to get 20 +80= 100]

I: OK Twenty plus eighty equals a hundred. Is there another way you can write it? Do you need another piece of paper?

S: yeah.[writes 100 – 20 =80]

I: So what is that sentence?

S: A hundred minus twenty equals 80

(3b) Student made connections between the links as units and links as groups.

I: Why do you think seventy or how did you figure that out?

S: Um… [Stretching the chain in their hands] Because thirty plus seventy equals one hundred and seven plus three equals ten so it'd have to equal one hundred.

Graph task:

(1) Connects the bars on the graph with notion of doubles.

S: because i knew that six plus six equals twelve and I added one more and it equals thirteen.

I: um, can you write a number sentence for that?

S: [writes 7+6=13]

CONTENT INDICATORS	**YES**	**NO**	**NO EVIDENCE**
1. Can student count objects by tens? (Links)	X		
2. Can student solve addition and/or subtraction problems involving multiples of ten? (Links)	X		
3. Does student make connections between basic addition facts for ten and multiples of ten? (Links)	X		
4. Can the student count on to solve addition problem situations? (Cubes)			X
5. Can a student count on or back to solve a subtraction situation? (Cubes)			X
6. Can the student read data from bar graphs? (Graph)	X		
7. Can the student use data to solve problems? (Graph)	X		
8. Can the student represent numbers using manipulatives and number sentences? (Links, Graph, Cubes)	X		

Math Trailblazers Research and Revision Project
Appendix D
Fraction and Proportionality Study Documents

University of Minnesota

Table 1: Fraction Test Summary

1. Understanding of fractions as parts of unit wholes, as parts of a collection, as locations on number lines, and as divisions of whole numbers

Five Parts to this:
a) Flexible choice of unit
b) Continuous (area or length) model (parts of unit wholes)
c) Discrete model (parts of a collection)
d) Number line (locations on number lines)
e) Division of whole numbers

	Item Number	4 n = 181	5 n = 154
a	4	29.8%	35.1%
	25	27.6%	39.0%
	26	28.7%	53.2%
b	5	10.5%	24.0%
	12	13.8%	32.5%
	32	83.4%	83.8%
c	6	8.3%	26.6%
	7	29.3%	56.5%
d	28	9.4%	14.3%
e	19	12.2%	12.3%

2. Use models, benchmarks, and equivalent forms to judge the size of fractions

Four student-constructed strategies for ordering fractions
a) Residual
b) Transitive
c) Same numerator
d) Same denominator
e) Other

	Item Number	4 n = 181	5 n = 154
a	13	49.2%	61.0%
b	14	57.5%	68.2%
	17	62.4%	68.2%
c	16	59.1%	63%
d	15	72.9%	82.5%
e	29	72.7%	79.6

3. Recognize and generate equivalent forms of commonly used fractions
a) Discrete model
b) Continuous model

	other			a	b	
	11	18	34	10	9	33
4	23.8%	26.0%	45.3%	22.1%	19.9%	35.9%
5	41.6%	36.4%	54.5%	42.2%	39.6%	48.7%

4. Develop and use strategies to estimate computations involving fractions

	1	2	3
4	32.6%	24.9%	21.5%
5	40.9%	30.5%	28.6%

5. Use visual models, benchmarks, and equivalent forms to add and subtract commonly used fractions

	20	21	22	23	24
4	17.1%	21.0%	51.9%	9.9%	15.5%
5	31.8%	24.7%	58.4%	25.3%	35.7%

6. Beyond PSSM:
a) Reconstructing the unit
b) Multiplication of fractions
c) Division of fractions

	Item Number	4 n = 181	5 n = 154
a	27	44.8%	68.8%
	30	11.6%	40.9%
	31	4.4%	29.2%
b	35	16.0%	21.4%
	36	9.9%	14.9%
	37	14.9%	17.5%
c	8	21.0%	30.5%

Sample Test Item Analysis

1. Estimate the answer by recording in the box the whole number the answer is closest to:

$$\frac{7}{8} + \frac{12}{13}$$

Source	RNP 1995
NCTM Goal	4. Develop and use strategies to estimate computation involving fractions
Rationale	This item will be flashed on the screen for 30 seconds so that the children will use their estimation abilities rather than the standard algorithm. Children who have a strong mental image of these fractions may notice that 7/8 is close to 1 and 12/13 is also close to 1 so the sum will be close to 2.

RNP Study 1995 Question 1			
	correct responses	# of classrooms	n
RNP	78%	33	839
AW	58%	27	827
HBJ	74%	6	
Inv5A	35%		660
Inv5B	58%		113
TB4	33%	9	181
TB5	41%	15	154

Table 2:
Math Trailblazers' Proportionality Written Test

1.	Simple Multiplication Problems: Given a unit rate (2 problems) Items 1 - 2		1	2
		Chicago (21)	57%	86%
		Dubuque (39)	80%	90%
		SLP (89)	80%	90%
2.	Simple Division Problems: Find the unit rate (2 problems) Items 3 - 4		3	4
		Chicago (21)	67%	38%
		Dubuque (39)	60%	54%
		SLP (89)	62%	53%

3.	Missing Value Problems (11 problems) a) Integer both Between and Within : 5 b) Integer Between: 8 c) Integer Within: 6, 7 d) No Integer Relationships: 9, 10 e) Using a Table: 11, 12, 13 f) Using a Graph: 14, 15 Items 5 – 15		5	6	7	8	9	10
		C 21	52%	52%	67%	48%	14%	5%
		D 39	28%	54%	21%	46%	13%	18%
		S 89	40%	67%	30%	62%	25%	15%
			11	12	13	14	15	
		C 21	91%	48%	48%	81%	86%	
		D 39	85%	38%	49%	82%	74%	
		S 89	90%	53%	58%	79%	80%	

4.	Numerical Comparison Problems (5 a mix of buying contexts and speed) Items 16 - 20		16	17	18	19	20
		C 21	62%	86%	29%	43%	71%
		D 39	69%	85%	33%	46%	62%
		S 89	69%	88%	43%	58%	81%

5.	Qualitative Compare and Predict Problems (4 problems) Items: 21 - 24		21	22	23	24
		C 21	71%	62%	71%	62%
		D 39	72%	74%	67%	79%
		S 89	69%	73%	60%	69%

6.	Large Problems (3 problems; science, math and real world contexts) Items: 25 - 33		25	26	27	28	29
		C 21	19%	43%	52%	38%	19%
		D 39	31%	38%	74%	54%	28%
		S 89	20%	24%	49%	44%	16%

Math Trailblazers Research and Revision Project
Appendix E
Grade 1 Revisions Documents

University of Illinois at Chicago
IMSE

Grade 1 Number and Operation Assessment Indicators by "Big Idea"				
Big Idea #	**PSSM Big Idea: MTB Assessment Indicators**		**2nd ed Unit**	**4th ed Unit**
1	**Count with understanding: Group and count objects (by 1s, 2s, 5s, 10s, & 100s)**			
1		Count objects	1	1
1		Count on from a given number	1	1
1		Identify the number of a small group of objects without counting	1	1
1		Count on to solve addition problems	3	3
1		Count on to solve addition problems	4	4
1		Divide a collections of objects into groups of a given size and count the leftovers	5	5
1		Group and count objects by twos, fives, and tens	5	5
1		Group and count objects by fives and ones	6	6
1		Count up or count back to solve subtraction problems	8	9
1		Count objects by twos, fives, and tens	9	10
1		Group and count objects by tens and ones	9	10
1		Group and count objects by fives and tens	11	11
1		Group and count objects by hundreds, tens, and ones	17	16
2	**Develop initial understanding of place value**			
2		Identify a number represented on a ten frame	3	3
2		Divide a collections of objects into groups of a given size and count the leftovers	5	5
2		Group and count objects by twos, fives, and tens	5	5
2		Group and count objects by tens and ones	9	10
2		Represent two-digit numbers using manipulatives, ten frames, and 100 Charts	9	10
2		Group and count objects by fives and tens	11	11

2		Represent two-digit numbers using ten frames, 100 Charts, manipulatives, and number sentences	11	11
2		Group and count objects by hundreds, tens, and ones	17	16
2		Represent numbers greater than 100 using manipulatives, symbols, and words	17	16
3	**Develop number sense and flexibility: Compare numbers; Partition numbers (small, 10, 2-digit, 100)**			
3		Compare numbers using more, less, or about the same	1	1
3		Partition a number into two and three parts	3	3
3		Partition numbers into two and three parts and represent them with number sentences	4	4
3		Divide a collections of objects into groups of a given size and count the leftovers	5	5
3		Describe a number in relations to other numbers	9	10
3		Partition 100 into groups of tens	11	11
3		Partition ten into two and three parts	13	12
4	**Translate between representations: Represent numbers (small, 2-digit, >100, fractions) using manipulatives, symbols, words, 10-frames, 100 Charts, and number sentences**			
4		Identify the number of a small group of objects without counting	1	1
4		Identify a number represented on a ten frame	3	3
4		Translate between representations of numbers (ten frames, tallies, manipulatives, and symbols)	3	3
4		Write number sentences for addition situations	4	4
4		Represent patterns using manipulatives, words, and symbols	7	7
4		Represent subtraction situations using whole-part-part language	8	9
4		Write number sentences for subtraction situations	8	9

4		Represent two-digit numbers using manipulatives, ten frames, and 100 Charts	9	10
4		Represent two-digit numbers using ten frames, 100 Charts, manipulatives, and number sentences	11	11
4		Represent multiplication and division situations using manipulatives or drawings	14	14
4		Represent numbers greater than 100 using manipulatives, symbols, and words	17	16
4		Represent and describe fractions (1/2 and 1/4) using manipulatives, drawings, and symbols	18	17
5	**Develop concepts of addition and subtraction along wiith strategies for computation: Solve addition and subtraction problems and explain their reasoning**			
5		Count on to solve addition problems	3	3
5		Solve addition problems and explain their reasoning	3	3
5		Use manipulatives to solve problems	3	3
5		Create a story for an addition number sentence	4	4
5		Partition numbers into two and three parts and represent them with number sentences	4	4
5		Solve addition problems and explain their reasoning	4	4
5		Write number sentences for addition situations	4	4
5		Use data to solve problems involving length	6	6
5		Count up or count back to solve subtraction problems	8	9
5		Create a story for a subtraction number sentence	8	9
5		Represent subtraction situations using whole-part-part language	8	9
5		Solve subtraction problems and explain their reasoning	8	9
5		Write number sentences for subtraction situations	8	9
5		Use data to solve problems involving volume	9	10

5		Solve addition and subtraction problems using multiples of five and ten	11	11
5		Solve addition and subtraction problems and explain their reasoning	13	12
5		Use doubles to solve addition problems	13	12
5		Use data to solve problems	14	14
5		Use data to solve problems	16	18
5		Solve addition problems using multiples of ten and 100	17	16
5		Solve addition and subtraction problems and explain their reasoning	20	20
5		Use data to solve problems	20	20
6	**Develop fluency with addition and subtraction math facts: Use math facts strategies to add**			
6		Use math facts strategies to add (direct modeling, counting strategies, or reasoning from known facts)	11-20	
7	**Understand multiplication and division situations: Solve multiplication and division problems and explain their reasoning**			
7		Divide a collections of objects into groups of a given size and count the leftovers	5	5
7		Create stories for multiplication and division situations	14	14
7		Represent multiplication and division situations using manipulatives or drawings	14	14
7		Solve multiplication and division problems and explain their reasoning verbally	14	14
7		Solve multiplication and division problems and explain their reasoning	20	20

8	**Identify the relationships among and find the value of a collection of pennies, nickels, and dimes (MTB ONLY)**			
8		Identify the relationships among pennies, nickels, and dimes	5	5
8		Find the value of a collection of nickels, dimes, and quarters	11	11
9	**Understand and represent commonly used fractions, such as 1/4, 1/3, and 1/2**			
9		Partition sets of objects into fractional parts	18	17
9		Recognize that fractional parts of a whole (halves and fourths) must have equal areas	18	17
9		Represent and describe fractions (1/2 and 1/4) using manipulatives, drawings, and symbols	18	17

Grade 1 Revision Framework: Proposed Revisions by Unit									
2nd ed Unit	***2nd ed Unit Name***	***2nd ed min # sess -ions***	***2nd ed max # sess- ions***	**4th ed Uni t**	***Remove from curiculum***	***Add to curriculum***	***Connections***	***4th ed min # sess- ions***	***4th ed max # sess -ions***
1	*Welcome to First Grade: A Baseline Assessment Unit*	8	8	1	Bubble Sort	Introduce number lines.	Connect counting and counting on w/ cubes to number lines	9	10
2	*Exploring Shapes*	8	8	2		Revise based on consultant notes.		8	8
3	*Pennies, Pockets, and Parts*	15	15	3	Replace L3 Think and Spin w/ more 10-frames work.	Perhaps L2 uses ten frames <=10. L3 includes numbers >10	Connect ten frames representations to number lines, graphs, pockets, & pennies work.	15	15
4	*Adding to Solve Problems*	5	5	4	Move odd & even to patterns unit? Delete counting-on cards as "not enough pay-off?"			6	6
5	*Grouping and Counting*	9	9	5	Make Adventure Book (AB) optional. Number strips are too much stuff--> use links?	Add "likely, unlikely, possible, impossible" to Colors lab?	Connect representations: 10 frames to 100 Charts to graphs.	9	10
6	*Measurement: Length*	10	10	6	Delete L5 Delightful Dachshunds? Make AB optional.	Revise based on feedback. More ruler, less links. Tie ruler to number line? Change unit title to include "counting". Add "likely, unlikely, possible, impossible" to lab.	Tie intervals in L2 to 100 Chart.	8	9

7	*Patterns and Designs*	5	7	7		In L3 Name Patterns, tie to 100 Chart. Include Odd and Even Lesson from old U4 and tie to 100 Chart. Include numbers in patterns and include number patterns such as multiples on 100 Chart, counting backwards. Where to put "growing patterns"? Does symmetry go here?	Patterns on 100 Chart.	7	7
10	*Measurement: Area*	5	5	8	Move work w/ halves to Unit 18. Delete AB Midnight Visit or change to match new lessons.	L1 & L2 okay. Change L2, 3, & 5 from work with halves to "counting on to add" and "addition w/ number sentences". Develop figures that students can count by 2s, 3s?, 5s, & 10s, and count on. Write number sentences for two or three parts of the figures. Make figures w/ given areas. Put two figures together w/ given areas. Find the total area. Include addition in unit title.	Connect to counting on the 100 Chart and then counting on, i.e. for a shape w/ 4 rows of 5 tiles and 1 row w/ 3, students can count: 5, 10, 15, 20, 21, 22, 23.	5	5
8	*Subtract to Solve Problems*	5	6	9	Delete clown and subtraction cartoons from Lesson 3.	Include number line work.	More connections to addition using part-part-whole model. More connecitons to 10-frame	6	6
9	*Grouping by Tens*	11	12	10	Use cubes instead of beans for L1? Remove "Spin for Beans" or change to marking 10 frames w/ x's.	Need another day for L1. Find another way to connect 10-frames and grouping by tens other than "Spin for Beans," Add a midyear review and/or test.	Make connections among representations	13	13

11	*Looking at 100*	11	11	11	Delete dividing chains into 4 parts in L1. L5 How Long is 100? may be problematic.	Replace L5 How Long is 100? with a Could Be or Crazy estimation lesson?	Connect chains in L1 to 100 Chart.	11	11
13	*Thinking About Addition and Subtraction*	8	8	12	Delete Doubles Railroad game in Part 3 of L3, use smaller numbers, or make an extension.	Tie to fact work in DPPs. Include addition w/ zero & commutative property. Connection between addition and subtraction. Include missing addend problems. More word problems. Include number lines as tool.	Make connections between strategies and tools in problem solving.	8	8
12	*Cubes and Volume*	6	7	13		Use towers to introduce multiplication. Add multiplication to unit title.	Connect to skip counting.	7	7
14	*Exploring Multiplication and Division*	5	8	14	Delete L1 Math Mice or change to drawing mice.	Add "likely, unlikely, possible, impossible" to pets lab.	Encourage use of a variety of tools and representations. Make connections amont strategies and representations.	5	8
15	*Exploring 3-D Shapes*	5	5	15	Delete L2 Sizing Cylinders.	Revise using teacher feedback and consultant recommendations. Add a day to L1.		5	5
17	*Moving Beyond 100*	5	6	16	Deal w/ "Too much stuff" issue.	Add more work w/ place value and work 10s & 100s? Add problems w/ sums > 20 solving w/ invented strategies to add & subtract. Change title. Add "could be or crazy". Use 2 100 Charts to go beyond 100?	Encourage use of a variety of tools and representations. Make connections among strategies and representations.	5	6
18	*Pieces, Parts, and Symmetry*	6	7	17	Deal w/ "Too much stuff" issue cutting and pasting in L1.	Introduce halves here w/ area form 2nd edition Unit 10? Use number line model for fractions?		6	7

16	*Collecting and Organizing Data*	5	5	18	Make optional or include more word problems for practice using food context.	Add word problems w/ sums greater than 20 to solve w/ invented strategies? Add "likely, unlikely, possible, impossible" to lab.		5	5
19	*Measurement and Mapping*	4	5	19	Make L2 more teacher friendly.			4	5
20	*Looking Back at First Grade*	4	7	20				4	7
		140	*154*					*146*	*158*

Math Trailblazers Research and Revision Project
Appendix F
Samples of Revised Grade 1 Materials

University of Illinois at Chicago

1. Unit 3 Lesson 5 *What's in That Pocket* field test version
2. Unit 3 Lesson 6 *What's in That Pocket* 2nd edition
3. Sample Home Practice pages

Grade 1 **Unit 3 Lesson 5** ***What's in That Pocket?*** **field test version**

Daily Practice and Problems

M. Calendar Count 5 (URG p. 12)

1. What are the names of the months?
2. How many months are in one year?

N. Calendar Count 6 (URG p. 13)

1. What is the first month of the year?
2. What is the last month of the year?

O. Calendar Count 7 (URG p. 13)

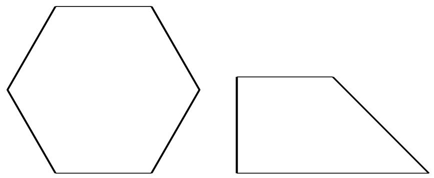
1. In which month is your birthday?
2. Find another student who was born in the same month as you.

P. What Do You Know About Rhombuses? (URG p. 13)

List all the things we know about rhombuses.

Q. Alike and Different 2 (URG p. 14)

1. What are these two shapes? How are they alike?
2. How are they different?



R. Tallies (URG p. 15)

T.J. has a shell collection. He listed all the colors of his shells: white, gray, white, tan, gray, gray, pink, gray, tan, pink, pink, gray, white, and gray. Use tallies to record the colors of T.J.'s shells in the table below.

Color	Tallies	Number
Pink		
Gray		
White		
Tan		

Then, write the number of shells for each color.

Suggestions for using the DPPs are on page 68.

LESSON GUIDE 5

What's in That Pocket?

Estimated Class Sessions: 3

Students partition the number ten into two parts. They practice filling ten frames and writing number sentences.

Key Content

- Translating between different representations of a number (ten frames, manipulatives, number lines, and symbols).
- Partitioning a number into two parts.
- Writing number sentences for addition situations.
- Organizing and analyzing data in a table.
- Solving addition problems and explaining strategies.
- Using manipulatives to solve addition problems.
- Reading and writing numbers to 20.

Curriculum Sequence

Before This Unit

Students regularly partitioned numbers in Kindergarten with particular emphasis on the numbers 5 and 10. See lessons in the Number Sense Strand in Months 3, 4, 7, and 8.

After This Unit

Students will continue to partition numbers throughout first grade as a way to develop math facts strategies. Students partition numbers 0–100 in Units 5, 8, 11, and 13.

Materials List

Print Materials for Students

		Daily Practice and Problems	Activity	Homework	Assessment
Student Book	**Student Guide**		*Two Pockets Work Mat* Page 61, *Pockets and Ten Frames* Pages 63–64, and *Five Pennies Data Table* Page 65	*Eight Pennies Data Table* Page 67	*Nine Pennies Data Table* Page 66
Teacher Resources	**Unit Resource Guide**	DPP Items M–R Pages 12–15	*Three Pockets Work Mat* Page 72, optional and *Three Pockets Data Table* Page 73, optional		

Supplies for Each Student

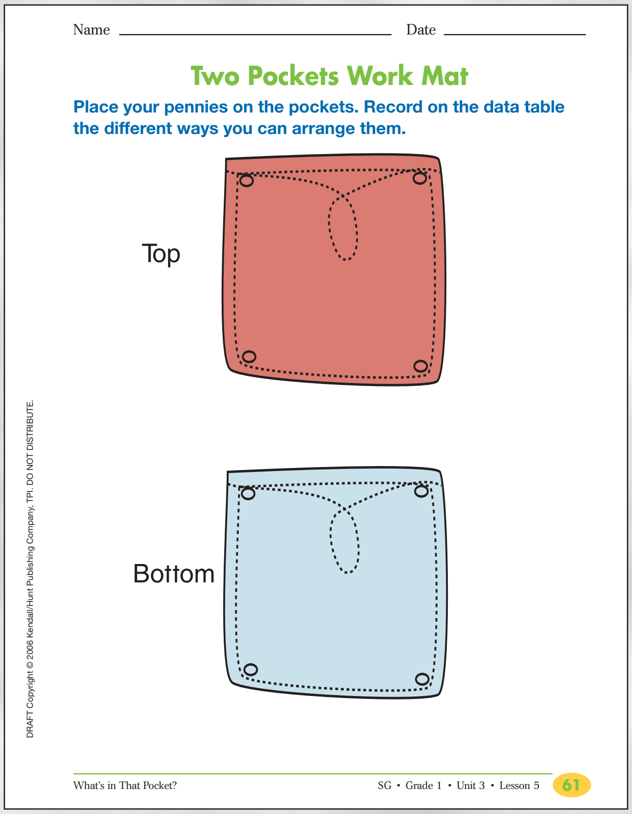
10 pennies

Materials for the Teacher

7 copies of *Ten Frames and Number Sentences* Blackline Master (Unit Resource Guide) Page 71
Observational Assessment Record (Unit Resource Guide) Pages 7–8
10 pennies
blank transparency

		Total Pennies	Number Sentence
6	4	10	6 + 4 = 10

Figure 8: *Data table for recording partitions of ten*



***Student Guide* - Page 61**

TIMS Tip

If necessary, remind students that putting zero pennies in the top pocket or zero pennies in the bottom pocket is a possibility.

Before the Activity

In a discussion in Part 1, the class will discuss the different ways to partition 10 into two parts. Create a four-column data table with 11 rows on chart paper or the board as shown in Figure 8. Make seven copies of the *Ten Frames and Number Sentences* Blackline Master. Cut the pages as shown so that you have fourteen ten frames with blanks for number sentences.

If you choose to lead the discussion using the overhead projector, make transparencies of the *Two Pockets Data Table* Transparency Master and the *Pockets and Ten Frames* Activity Pages in the *Student Guide.*

Developing the Activity

Part 1. Partitioning Ten into Two Parts

The *Two Pockets Work Mat* in the *Student Guide* has a picture of a pocket at the top and at the bottom of the page. Using a transparency marker, sketch two pockets on a transparency. Make the pockets large enough to fit ten pennies. Draw a ten frame alongside the pockets large enough to fit pennies in the boxes.

Tell students they are going to put their ten pennies into two pockets on their *Two Pockets Work Mat.* Place six pennies in the top pocket and four pennies in the bottom pocket as an example. Ask:

- *How many pennies are in the top pocket?* (6)
- *How many pennies are in the bottom pocket?* (4)
- *What number sentence can I write to describe my pennies?* (6 + 4 = 10)
- *Is this the only way I can divide the ten pennies in my two pockets? Show me another way.*

Encourage students to show other ways to divide the pennies and write corresponding number sentences. Since there is more than one way to solve this problem, tell them they will keep track of all the different ways on the *Pockets and Ten Frames* Activity Pages. They will record the different solutions on ten frames and in number sentences as shown in the example.

Ask a student to come to the overhead and share his or her solution. After the child places the pennies in the pockets, ask the class if this is a different solution. Students should record the solution at their desks to practice filling in the ten frames.

Come to an agreement that solutions where the pennies are reversed in the two pockets (e.g., four pennies in the top and six in the bottom when a previous entry had six pennies in the top and four in the bottom) are two different solutions. Ask:

- *What happens to the total number of pennies when the numbers of pennies are turned around?* (The total is the same.)

Have students work in pairs to find as many different ways as they can to divide the ten pennies between the pockets and to record their solutions on the *Pockets and Ten Frames* Activity Pages. Students should follow the example and place Xs in the ten frame to represent the pennies in the top pocket and dots to represent those in the bottom pocket. When appropriate, challenge student pairs to see if they have all the possibilities.

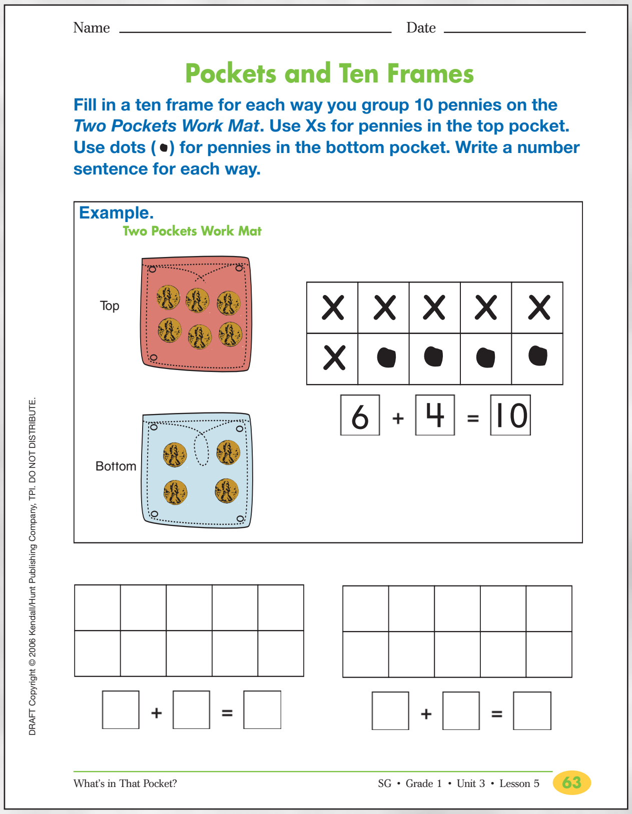
Part 2. How Many Ways?
Bring the class together and ask student pairs to share their solutions. Ask one student in the pair to place pennies in the pockets on the overhead and record their solution on a ten frame and in a number sentence. Use the strips you prepared from the *Ten Frames and Number Sentences* Blackline Master or a transparency of the Activity Page. As each solution is presented, tape the strip to the board or display it on the overhead and ask the class to check to make sure that the number sentence represents the combination of pennies and that it is a new solution. Ask:

- *Is this a different solution from any that are already posted? Is this a different ten frame and a different number sentence? Tell me how you know.* (Possible responses: It is already up on the board. See we already have a ten frame with just four Xs. Or, it is already there. It is 4 + 6 = 10 and John put up 4 + 6 = 10 for the first one.)

If it is a new solution, then post it with the other strips with ten frames and number sentences. If it is a duplicate, post it in a different place on the board with strips labeled "Repeats."

After students share their solutions, present this challenge:

- *Do we have all the possible ways to arrange ten pennies into two pockets?*


Name Date

Pockets and Ten Frames

Fill in a ten frame for each way you group 10 pennies on the *Two Pockets Work Mat*. Use Xs for pennies in the top pocket. Use dots (●) for pennies in the bottom pocket. Write a number sentence for each way.

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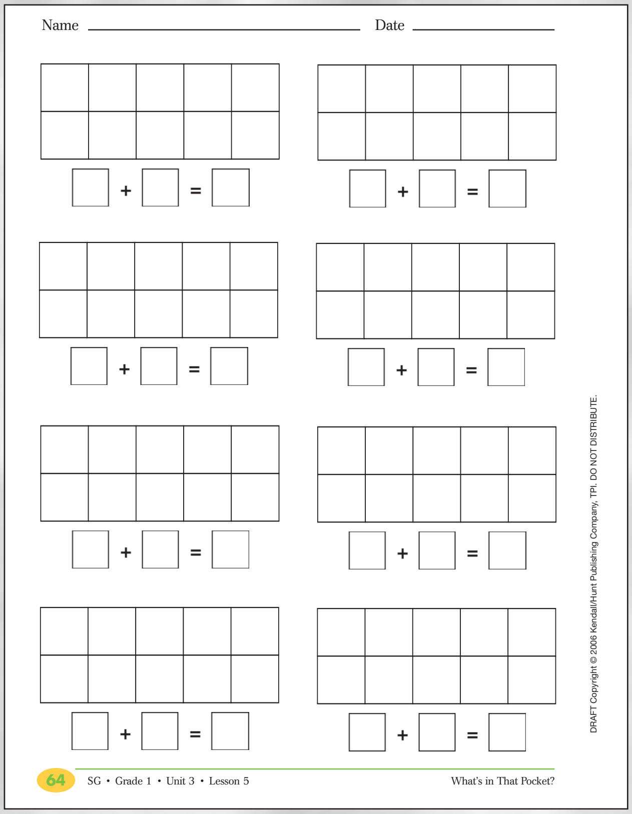
What's in That Pocket? SG • Grade 1 • Unit 3 • Lesson 5 63

***Student Guide* - Page 63**


Name Date

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64 SG • Grade 1 • Unit 3 • Lesson 5 What's in That Pocket?

***Student Guide* - Page 64**

A sample dialog follows:

Teacher: *Can anyone find a combination that is missing? Talk with your partner.*

Ana: *Jackie and I thought of 9 + 1 = 10.*

Teacher: *Is 9 +1 = 10 a different solution than those on the board?* [The class agrees that it is.] *How did you figure that out?*

Ana: *We saw 1 + 9 = 10, but 9 + 1 wasn't there.*

Teacher: *What makes 9 + 1 and 1 + 9 different?*

Ana: *There is one dot, but ours has nine dots.*

Teacher: *The ten frames do look different. You saw that you can turn the numbers around. The one on the board has nine Xs and one dot, but the ten frame Ana is describing has one X and nine dots. Ana, please make that ten-frame strip and put it on the board next to the one with nine Xs and one dot. Jackie, record the information in our table. Can someone find another missing combination?*

Nicholas: *How 'bout 10 + 0?*

Teacher: *How did you think of that?*

Nicholas: *When we did the pocket parts, sometimes a pocket had zero pennies.*

Teacher: *What will that ten frame look like?*

Nicholas: *10 Xs and no dots.*

Teacher: *What is the number sentence?*

Luis: *10 + 0 = 10. And turn it around 0 + 10 = 10.*

Teacher: *Good thinking.* [The students post the strips and record the information in the table. At this point, the class has filled in 10 ten frames and has found all but one solution as shown in Figure 9.] *Is there another missing combination?*

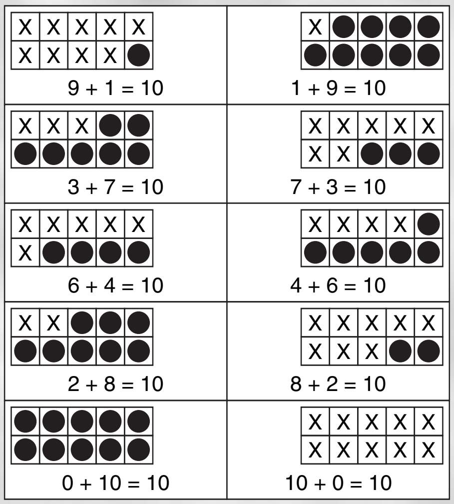


Figure 9: *The class arrangement of ten-frame strips*

Maya: *There aren't any.*

Teacher: *How can we tell if there are more missing combinations?*

Linda: *Well… we can't think of any more.*

Teacher: *How can we be sure that we have all of the combinations?*

Linda: *We have to check them.*

Teacher: *Explain what you mean by checking.*

Linda: *I think we have to try the numbers in the top.*

Teacher: *What numbers do you want to try?*

Linda: *The ones we used, you know—one, two, three, four, five, six, seven, eight, nine, ten.*

Luis: *We forgot zero again.*

Teacher: *If I start with zero pennies in the top pocket and ten pennies in the bottom, do I have that combination already?* [The teacher places ten pennies in the bottom pocket.]

Luis: *We just put that one up and its turn around.* [The teacher moves the ten-frame strip showing 0 + 10 = 10 to a place where she can make a new column as the class checks their work.] *Over here we can keep track of the ones we know we have tried. Where is this one in our table?* [Luis points to the table entry.]

Teacher: *Now, using Linda's strategy, I will put one penny in the top pocket and nine in the bottom and see if we have the combination.* [The teacher repeats the process with one, two, three, and four pennies and continues moving the strips into a column creating a visual pattern as in Figure 10. The combination with five pennies in each pocket is missing. The teacher places five pennies in the top pocket and five in the bottom.] *Do we have this combination already?*

Romesh: *No.*

Teacher: *Describe the ten-frame strip that matches.*

Romesh: *It would be five and five.*

Teacher: *What do you mean, "five and five?"*

Romesh: *Five Xs on top and five dots on the bottom.*

Teacher: *Great. Let's make a strip and add that solution to the table.* [The class continues the process until they have arranged all the strips as shown in Figure 10.]

There are eleven possible solutions as shown in Figure 10. You can also reorganize the data table so that the numbers in each column are in order as shown in Figure 11. Referring to the ten-frame strips, ask:

- *What patterns do you see in the ten frames and number sentences?* (Possible responses include: (1) The way you put the pennies shows on the board. First there are all dots and no Xs, then one X, then two Xs. It keeps going like that. (2) The number of Xs gets one bigger each time and the number of dots gets smaller. (3) The numbers in the number sentences match the Xs and dots. The first number gets one larger and the second number gets one smaller. (4) The first five number sentences are the turn-around sentences for the bottom five sentences.

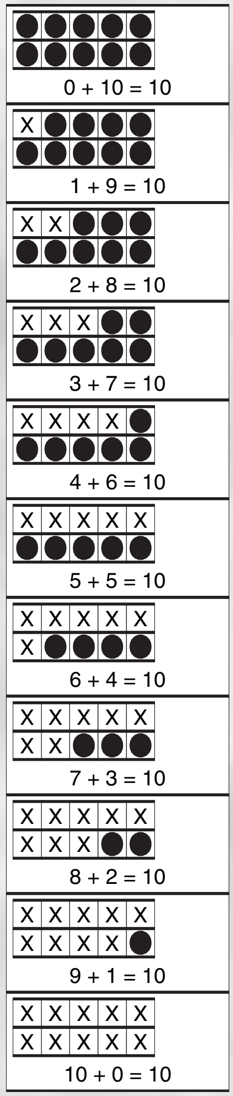


Figure 10: *The final arrangement of ten-frame strips*

		Total Pennies	Number Sentence
10	0	10	10 + 0 = 10
9	1	10	9 + 1 = 10
8	2	10	8 + 2 = 10
7	3	10	7 + 3 = 10
6	4	10	6 + 4 = 10
5	5	10	5 + 5 = 10
4	6	10	4 + 6 = 10
3	7	10	3 + 7 = 10
2	8	10	2 + 8 = 10
1	9	10	1 + 9 = 10
0	10	10	0 + 10 = 10

Figure 11: *Data table with eleven possible answers*

Name ______________________ Date ______________

Five Pennies Data Table

How many ways can you arrange five pennies in two pockets? Record as many ways as you can. Then, write a number sentence for each one.

		Total Pennies	Number Sentence
0	5	5	0 + 5 = 5

***Student Guide* - Page 65**

Name ______________________ Date ______________

Eight Pennies Data Table

Homework

Dear Family Member:

Your child will need eight pennies for this assignment. He or she should divide the pennies in different combinations between two pockets. The total should equal eight for each problem. Please help your child complete the data table.

Thank you.

How many ways can you arrange eight pennies in two pockets? Record as many ways as you can. Then, write a number sentence for each one.

		Total Pennies	Number Sentence
0	8	8	0 + 8 = 8

***Student Guide* - Page 67**

Summarizing the Lesson

Use the *Five Pennies Data Table* to provide further experiences partitioning numbers into two parts and writing corresponding number sentences. Encourage students to use their *Two Pockets Work Mat* and five pennies.

To bring the class to a close, discuss students' work partitioning five into two parts. Make connections between their work partitioning five with their work partitioning ten. Did they use the same strategies to find different ways to partition the five pennies?

Suggestions for Teaching the Lesson

Homework and Practice

- Assign the *Eight Pennies Data Table* Homework Page. Families will help their children find different ways to arrange eight pennies into two pockets. Students record the data in the table and write a number sentence for each arrangement.
- DPP items M–O provide practice using a calendar. DPP items P–Q review geometry concepts. Item R practices the use of tallies.

Assessment

- Students complete the *Nine Pennies Data Table* Assessment Page. Students must find different ways to place nine pennies into two pockets. Assess whether students can partition nine into two parts in more than one way and whether they can write corresponding number sentences. Encourage children to find as many ways as they can, but don't expect them to find all the ways.
- Record your observations on the *Observational Assessment Record,* noting students' progress in partitioning numbers and using symbols to write corresponding number sentences.

Use the following Assessment Indicators:

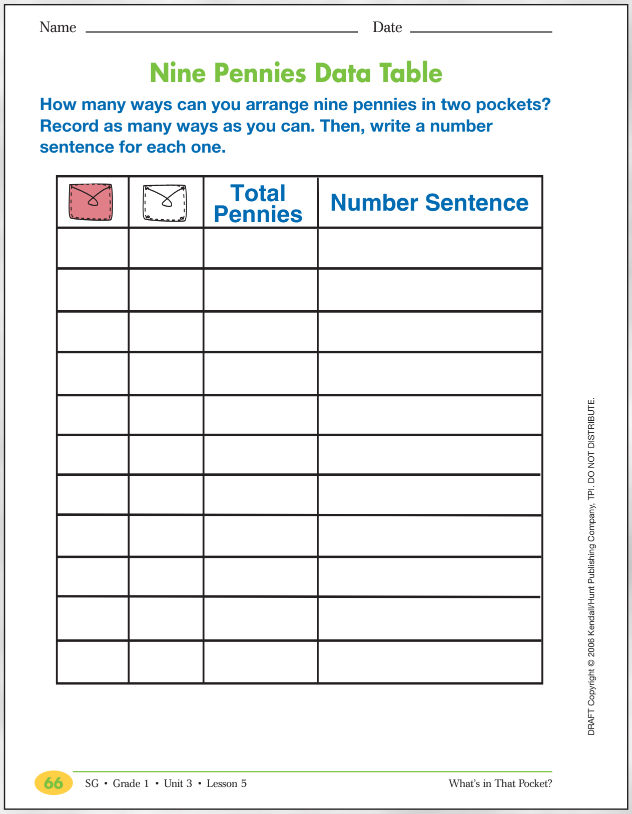
A2. Can students translate between representations of numbers (ten frames, tallies, manipulatives, number lines, and symbols)?

A4. Can students partition a number into two parts?

A6. Can students write number sentences for addition situations?

Extensions

- When students present their solutions at the overhead projector, ask them to place pennies only in the top pocket. The rest of the class must decide how many pennies should go in the bottom pocket, so that the total is ten. This is an introduction to missing addend problems.
- Develop a center activity in which students partition ten into three parts. Use the *Three Pockets Work Mat* and the *Three Pockets Data Table* Blackline Masters.


Name ______________________ Date ____________

Nine Pennies Data Table

How many ways can you arrange nine pennies in two pockets? Record as many ways as you can. Then, write a number sentence for each one.

		Total Pennies	Number Sentence

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66 SG • Grade 1 • Unit 3 • Lesson 5 What's in That Pocket?

***Student Guide* - Page 66**

AT A GLANCE

Math Facts Strategies and Daily Practice and Problems

DPP items M, N, and O provide practice using a calendar. DPP items P and Q review geometry concepts. DPP item R reviews use of tallies in data collection.

Part 1. Partitioning Ten into Two Parts A2 A4 A6

1. Sketch two pockets, one above the other, and a ten frame on an overhead transparency.
2. Students divide 10 pennies between the two pockets. They write corresponding addition number sentences and fill in ten frames appropriately.
3. Students work in pairs to find as many different ways to divide 10 pennies as they can and record their solutions on the *Pockets and Ten Frames* Activity Pages.

Part 2. How Many Ways? A2 A4 A6

1. Student pairs share their solutions. As each solution is given, it is displayed using the strips prepared from the *Ten Frames and Number Sentences* Blackline Master and on a data table.
2. Students check to make sure there are no duplicates. The class agrees that turn-around solutions (6 + 4 and 4 + 6) are separate solutions.
3. Students discuss how to determine whether all the possible solutions have been found and develop an organized list showing all possible solutions.

Summarizing the Lesson A6

Have students complete the *Five Pennies Data Table* Activity Sheet, using their *Two Pockets Work Mat* and pennies. Discuss students' work partitioning five into two parts.

Homework

Students complete the *Eight Pennies Data Table* Homework Page, using strategies discussed in class.

Assessment

1. Students complete the *Nine Pennies Data Table* Assessment Page.
2. Use the Assessment Indicators (A2, A4, A6) and the *Observational Assessment Record* to document students' abilities to partition numbers into two parts and write appropriate number sentences.

Notes:

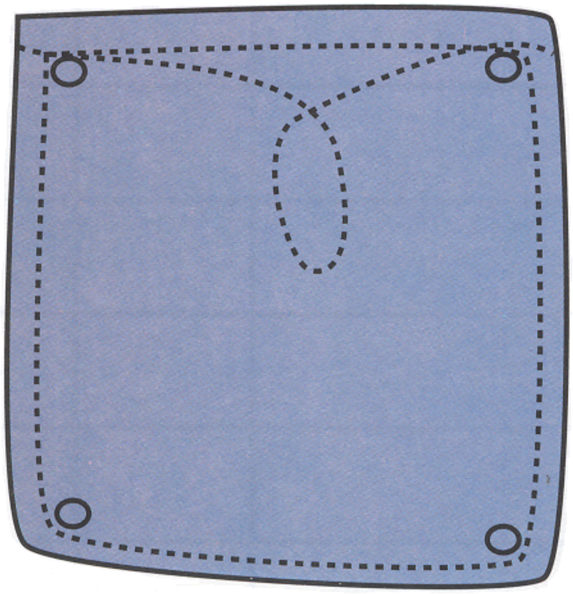
Ten Frames and Number Sentences

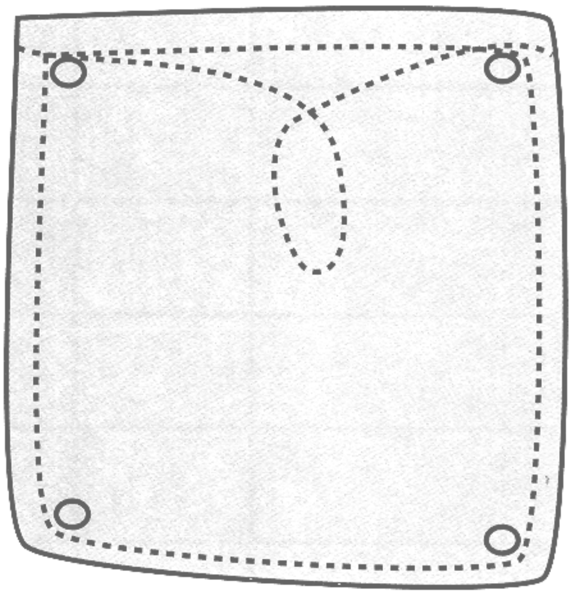
$$\square + \square = \square$$

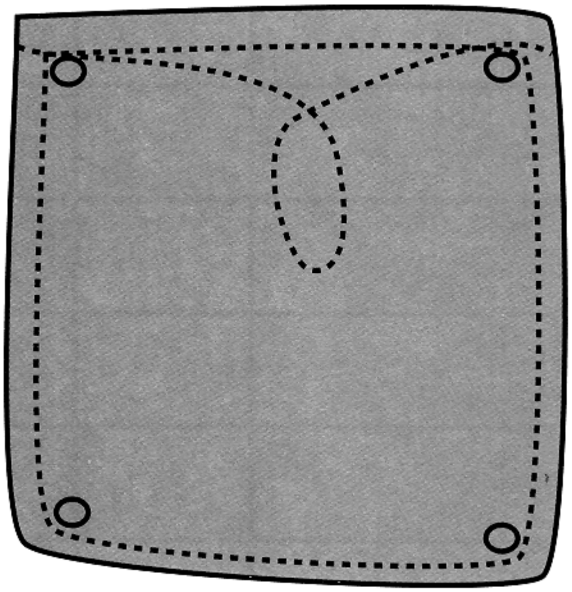
$$\square + \square = \square$$

Three Pockets Work Mat

Place your pennies on the pockets. Record on the data table different ways you can arrange them.

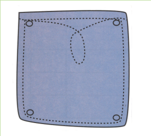
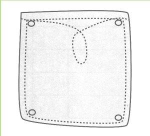
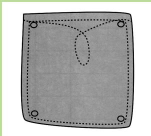






Three Pockets Data Table

How many ways can you arrange ten pennies in three pockets? Record as many ways as you can. Then, write a number sentence for each one.

			Total Pennies	Number Sentence
1	2	7	10	1 + 2 + 7 = 10

Grade 1 Unit 3 Lesson 6 *What's in That Pocket?* 2nd edition

LESSON GUIDE 6

What's in That Pocket?

Estimated Class Sessions: 3

Students partition the number ten into two and three parts. They practice writing number sentences and finding missing parts.

Key Content

- Translating between different representations of a number.
- Partitioning a number into two and three parts.
- Writing addition number sentences.
- Organizing and analyzing data in a table.
- Solving addition problems and explaining strategies.
- Using manipulatives to solve problems.

Daily Practice and Problems

W. What Number Once More? (URG p. 15)

1. What number follows fifteen?
2. What number comes just before fifteen?
3. What number follows nineteen?
4. What number comes just before nineteen?

X. Penny Problems 1 (URG p. 15)

You have ten pennies. Three pennies are in your shirt pocket. The rest are in your pants pocket. How many pennies are in your pants pocket?

Y. Penny Problems 2 (URG p. 15)

1. Sue found ten pennies in her pockets. Eight pennies were in her shirt pocket. How many were in her pants pocket?
2. Sue found eight pennies in her pockets. One penny was in a shirt pocket. Four pennies were in her pants pockets. How many were in her coat pocket?

Z. Tallies (URG p. 16)

T.J. has a shell collection. He listed all the colors of his shells: white, gray, white, tan, gray, gray, pink, gray, tan, pink, pink, gray, white, and gray. Use tallies to record the colors of T.J.'s shells in the table below.

Color	Tallies	Number
Pink		
Gray		
White		
Tan		

Then, write the number of shells for each color.

AA. Pennies (URG p. 16)

Emily has 6 pennies. Complete the table below to show different ways Emily can place her 6 pennies into 2 pockets. Then, write a number sentence for each.

Pocket 1	Pocket 2	Number Sentence
0¢	6¢	0¢ + 6¢ = 6¢

BB. More Pennies (URG p. 17)

Tyler has 5 pennies. Complete the table below to show different ways Tyler can place his 5 pennies into 3 pockets. Then, write a number sentence for each.

Pocket 1	Pocket 2	Pocket 3	Number Sentence
1¢	1¢	3¢	1¢ + 1¢ + 3¢ = 5¢

Suggestions for using the DPPs are on pages 53–54.

Curriculum Sequence

Before This Unit

Students regularly partitioned numbers in Kindergarten with particular emphasis on the numbers 5 and 10. See lessons in the Number Sense Strand in Months 3, 4, 7, and 8.

After This Unit

Students will continue to partition numbers throughout first grade as a way to develop math facts strategies. Students partition numbers 0–100 in Units 5, 8, 11, and 13.

Materials List

Print Materials for Students

		Daily Practice and Problems	Activity	Homework	Assessment
Student Book	**Student Guide**		*Two Pockets Work Mat* Page 55, *Two Pockets Data Table* Page 57, *Three Pockets Work Mat* Page 58, and *Three Pockets Data Table* Page 59	*Eight Pennies Data Table* Page 61	*Nine Pennies Data Table* Page 63
Teacher Resources	**Unit Resource Guide**	DPP Items W–BB Pages 15–17 (CD)			DPP Item Y *Penny Problems 2* Page 15 (CD)
	Generic Section (CD)		*Ten Frames,* 1 per student		

(CD) *available on Teacher Resource CD*

All Transparency Masters, Blackline Masters, and Assessment Blackline Masters in the Unit Resource Guide are on the Teacher Resource CD.

Supplies for Each Student

10 pennies

Materials for the Teacher

Observational Assessment Record (Unit Resource Guide, Pages 7–8 and Teacher Resource CD)
10 pennies
blank transparency

Before the Activity

Make a four-column and a five-column data table on the chalkboard, similar to the tables on the *Two Pockets Data Table* and the *Three Pockets Data Table* Activity Pages in the *Student Guide.*

Developing the Activity

Part 1. Partitioning Ten into Two Parts

The *Two Pockets Work Mat* in the *Student Guide* has a picture of a red and a blue pocket. Using a red and blue transparency marker, draw two pockets on a transparency. Make the pockets large enough to fit ten pennies.

Tell students they are going to put their ten pennies into two pockets using their *Two Pockets Work Mat.* Place six pennies in the red pocket and four pennies in the blue pocket as an example. Ask:

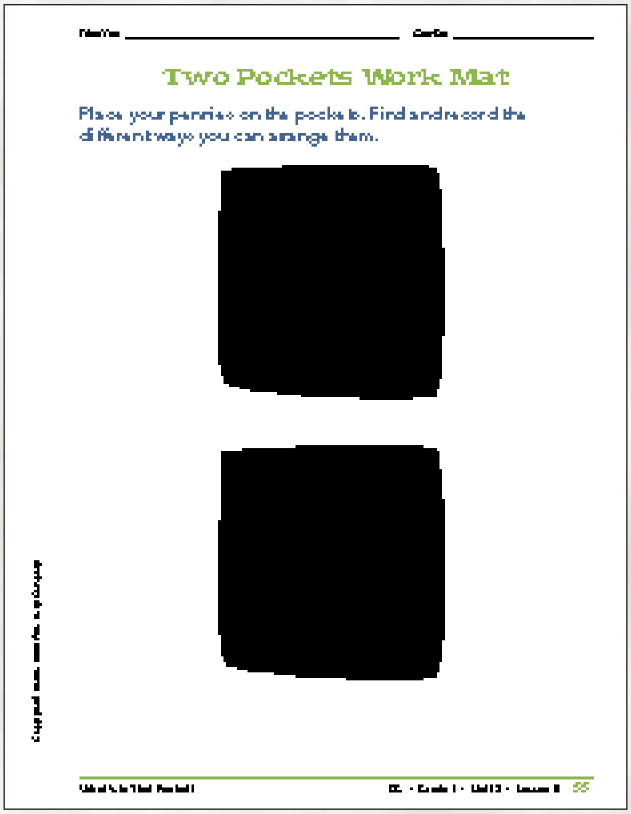
- *How many pennies are in the red pocket?* (6)
- *How many pennies are in the blue pocket?* (4)
- *What number sentence can I write to describe my pennies?* (6 + 4 = 10)
- *Is this the only way I can divide the ten pennies in my two pockets?*

Encourage students to show other ways to divide the pennies. Since there is more than one way to solve this problem, tell them they will keep track of all the different ways in a data table. Have students record the data, including the number sentence, on the *Two Pockets Data Table* Activity Page.

Ask students to come to the overhead and share their solutions. After each child places the pennies in the pockets, ask the class if this is a different solution. Encourage students to refer to the data table. If the solution does not already exist, record it. Students should also record each solution at their desks to practice filling in a data table.

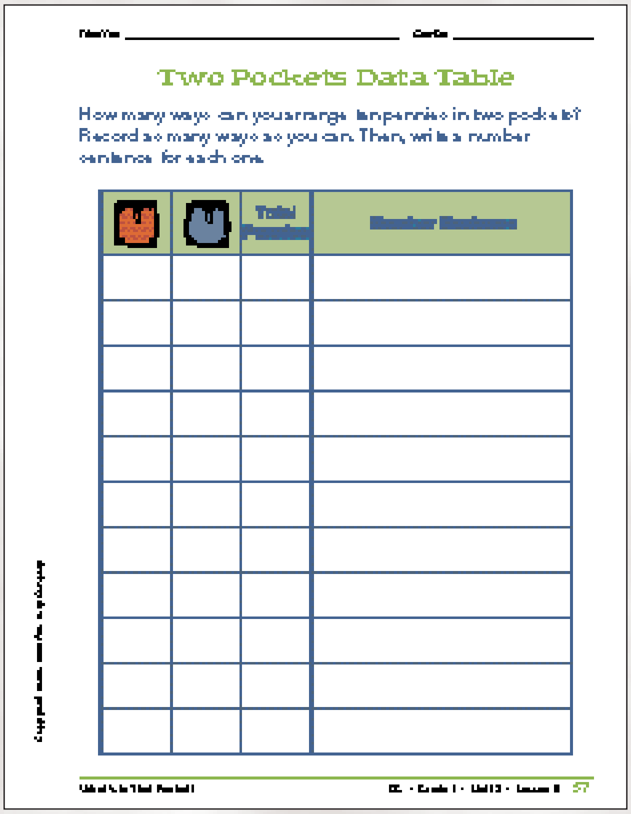
Solutions where the pennies are reversed in the two pockets (e.g., four pennies in the blue and six in the red when a previous entry had six pennies in the blue and four in the red) are two different solutions. Ask:

- *What happens to the total number of pennies when the numbers on the data table are turned around?* (The total is the same.)


Two Pockets Work Mat

Place your pennies on the pockets. Find and record the different ways you can arrange them.

***Student Guide* - Page 55**


Two Pockets Data Table

How many ways can you arrange ten pennies in two pockets? Record as many ways as you can. Then, write a number sentence for each one.

***Student Guide* - Page 57**

After students share their solutions, you might present a challenge:

- *Do we have all the possible ways to arrange the ten pennies into two pockets?*
- *How can we find out if we have all the possible answers?*

Encourage the class to work together to find all the possible answers. There are eleven possible solutions as shown in Figure 8. Ask pairs to report their solutions as you record them on the class data table. Reorganize the data table so the numbers in each column go in order. Ask:

- *What patterns do you see in the data table?* (Answers will vary. Students may notice that organizing the data will help show any missing answers. Students may notice that if the numbers in the first column derease by one, the numbers in the second column increase by one.)

TIMS Tip

Remind students that putting zero pennies in the red pocket

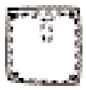
		Total Pennies	Number Sentence
10	0	10	10 + 0 = 10
9	1	10	9 + 1 = 10
8	2	10	8 + 2 = 10
7	3	10	7 + 3 = 10
6	4	10	6 + 4 = 10
5	5	10	5 + 5 = 10
4	6	10	4 + 6 = 10
3	7	10	3 + 7 = 10
2	8	10	2 + 8 = 10
1	9	10	1 + 9 = 10
0	10	10	0 + 10 = 10

Figure 8: *Data table with eleven possible answers*

Part 2. Partitioning Ten into Three Parts
This activity is similar to the previous one, except that children will use the *Three Pockets Work Mat* and the *Three Pockets Data Table.* Students find different ways to arrange ten pennies into three pockets. There are 66 ways of arranging the pennies if you allow one or two pockets to have no pennies. There are 36 possibilities if each pocket has at least one penny. Children should realize they are finding just a few of the possible answers. Encourage students to write number sentences with three parts and a total.

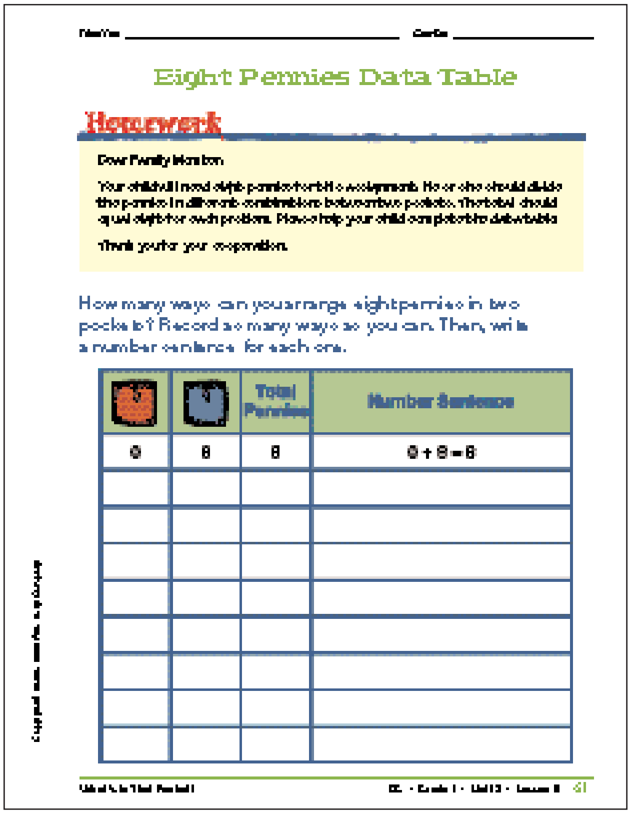
Suggestions for Teaching the Lesson

Math Facts Strategies

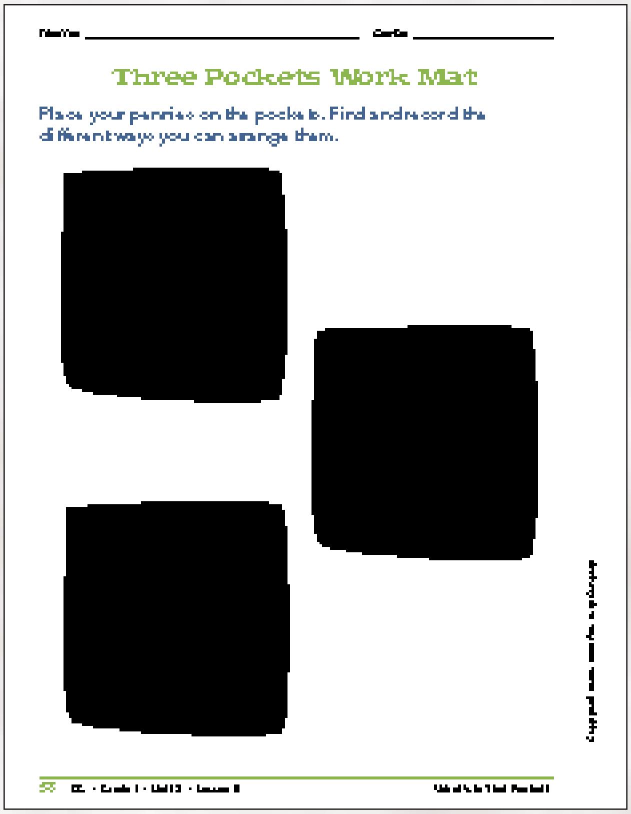
DPP items X, Y, AA, and BB review addition fact strategies in the context of money in pockets. DPP item W reviews counting patterns necessary for counting on and counting back.

Homework and Practice

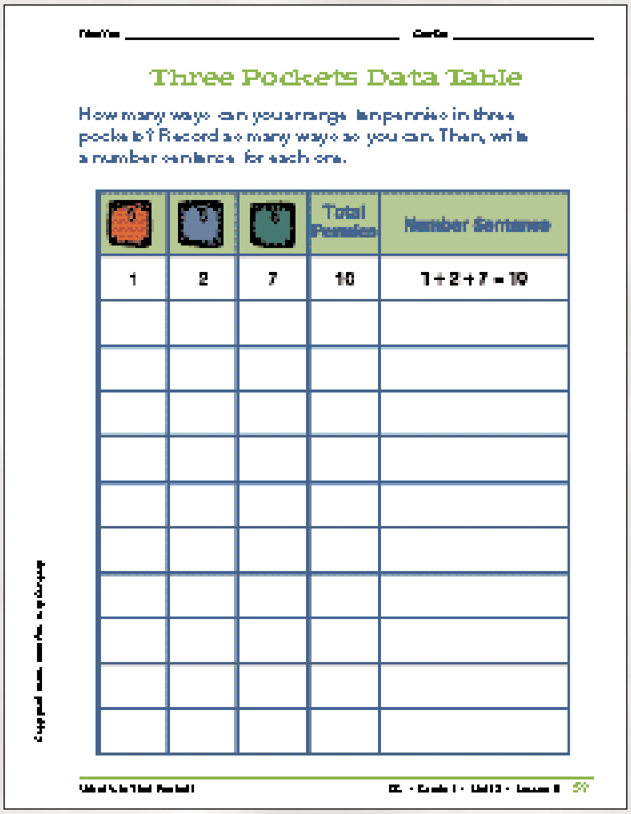
- Assign the *Eight Pennies Data Table* Homework Page. Families will help their children find different ways to arrange eight pennies into two pockets. Students record the data in the table and write a number sentence for each arrangement.
- DPP item Z practices using tallies.



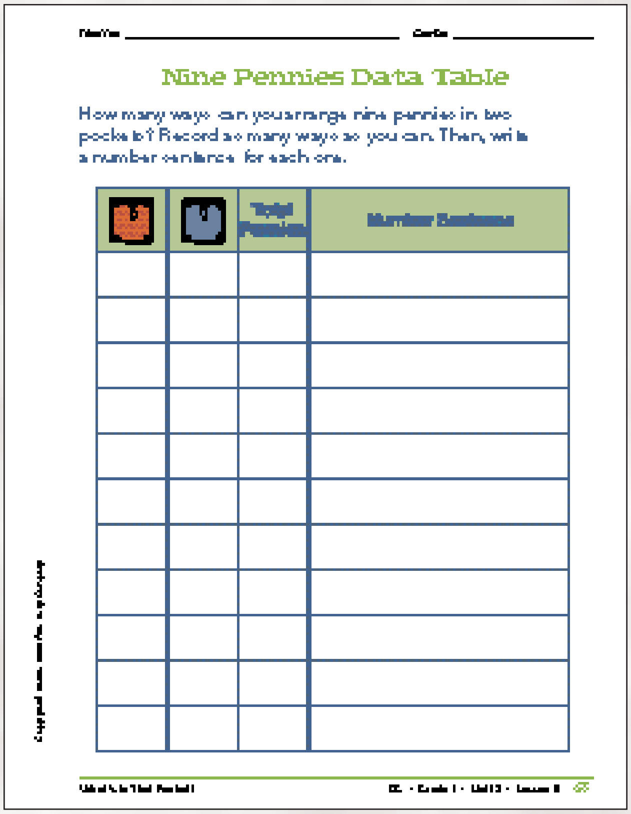
Student Guide - Page 61



Student Guide - Page 58



Student Guide - Page 59



Student Guide **- Page 63**

Suggestions for Teaching the Lesson *(continued)*

Assessment

- Students complete the *Nine Pennies Data Table* Assessment Page. Students must find different ways to place nine pennies into two pockets. Assess whether students can partition nine into two parts in more than one way and whether they can write a corresponding number sentence. Encourage children to find as many ways as they can, but don't expect them to find all the ways.
- Use DPP item Y as a performance assessment. Give students pennies and a copy of *Ten Frames* from the Generic Section to solve the problem.
- Record your observations on the *Observational Assessment Record,* noting students' progress in partitioning numbers and using symbols to write corresponding number sentences.

Extension

When students present their solutions at the overhead projector, ask them to place pennies only in the red pocket. The rest of the class must decide how many pennies should go in the blue pocket, so that the total is ten. This is similar to Part 2 of Lesson 5.

AT A GLANCE

Math Facts Strategies and Daily Practice and Problems

DPP items X, Y, AA, and BB review addition fact strategies. DPP item W reviews number patterns. DPP item Z practices using tallies.

Part 1. Partitioning Ten into Two Parts A2 A4

1. Draw a picture of a red pocket and a blue pocket on a transparency.
2. Make one four-column data table and one five-column data table on the chalkboard.
3. Tell students they are going to put ten pennies into two pockets using the *Two Pockets Work Mat* in the *Student Guide.*
4. Place six pennies in the red pocket and four pennies in the blue pocket. Write a number sentence (6 + 4 = 10).
5. Encourage students to show other ways to divide the pennies and write number sentences.
6. Students record their solutions in the *Two Pockets Data Table.*
7. Reorganize the data table so numbers in the blue pocket go in order from 0 to 10. Ask students to describe patterns they see.

Part 2. Partitioning Ten into Three Parts A2 A4

1. Draw a picture of a red pocket, a blue pocket, and a green pocket on a transparency.
2. Tell students they are going to put ten pennies into three pockets using the *Three Pockets Work Mat.*
3. Students record possible arrangements and write number sentences in the *Three Pockets Data Table.*

Homework

Assign the *Eight Pennies Data Table* Homework Page.

Assessment

1. Students complete the *Nine Pennies Data Table* Assessment Page.
2. Use DPP item Y as an assessment.
3. Use the Assessment Indicators (A2, A4) and the *Observational Assessment Record* to document students' abilities to partition numbers and translate their work into addition number sentences.

Notes:

Sample Home Practice Pages

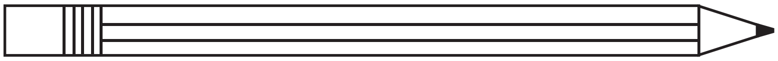
Name ______________________ Date ______________

Home Practice

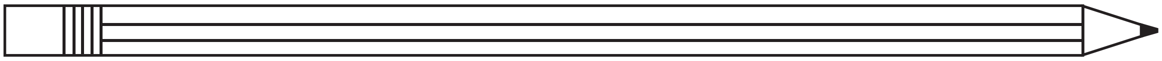
Part 3.

1. Measure the pencils in inches. Use your own ruler or cut out the ruler below.

A.



B.

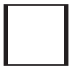


Pencil A is ____________ inches long.

Pencil B is ____________ inches long.

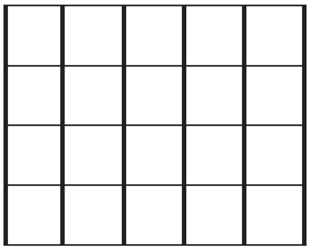
How much longer is B than A? ____________ inches

2. The shapes of rooms in a dollhouse are shown below.



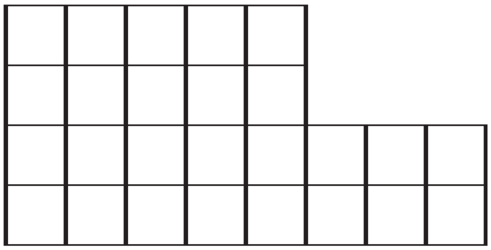
1 square unit

A.



Bedroom

B.

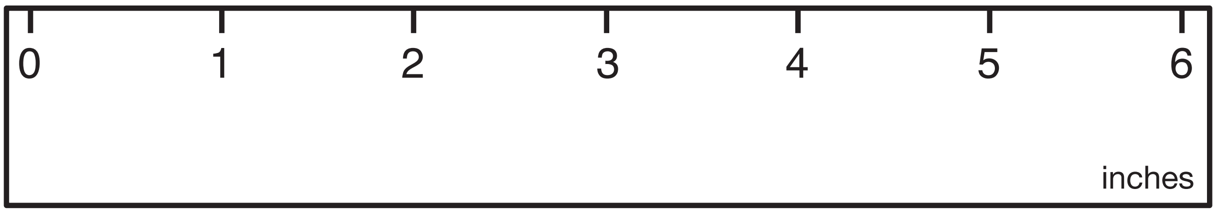


Kitchen

The area of the bedroom is ____________ square units.

The area of the kitchen is ____________ square units.

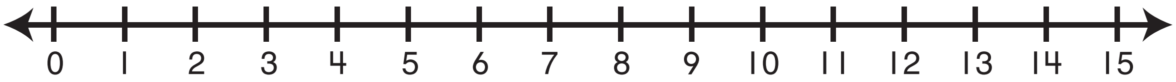
How much more area does the kitchen floor cover than the bedroom? ____________ square units.

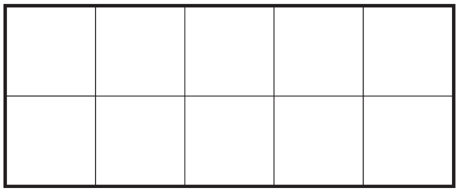


Name ______________________ Date ______________

Part 4.

Solve each problem. Write a number sentence for each. You may use the number line, ten frame, or counters such as pennies to help you. You may also draw a picture or diagram.





1. Ming had ten cookies in his lunch. He gave 3 to Jackie. How many cookies did he have left?

   ____

2. Maya has 9 stickers. Jacob has 6. How many more stickers does Maya have than Jacob?

   ____

3. Shannon and John combined their rock collections. Together they have 8 rocks. If Shannon had 5 rocks, how many rocks did John have?

   ____

4. Jerome picked 5 flowers for his teacher. He bought 4 more. How many did he give to her in all?

   ____